2

no p.f.

$$R(s) = \sum_{\sqrt{\frac{T}{24}} < n < T} n^{-s} + O(T^{-\sigma}).$$

Further $\psi(s) = \sum_{1}^{\xi} \frac{\mu(n)}{n^s}$

$$\xi(s)\psi(s) = P(s)\psi(s) + Q(s)\psi(s) + R(s)\psi(s)$$

$$P(s)\psi(s) = 1 + \sum_{T^{\frac{1}{L}}}^{T^{\frac{1}{\ell}}} \frac{a_n}{n^s} = 1 + \sum_{i=\ell+1}^{L} u_i(s)$$

where $u_i(s) = \sum_{\xi_i < n \leq \xi_{i-1}} \frac{a_n}{n^s}$, $\ell+1 \leq i \leq L$

$$T^{\frac{1}{\ell}} = \xi_\ell > \xi_{\ell+1} \quad \cdots \quad > \xi_L = T^{\frac{1}{L}}$$

$$Q(s)\psi(s) = \sum_{i=1}^{\ell} N_i(s), \text{ where } i = 1, \ldots \ell$$

$$u_i(s) = \psi(s) \cdot \sum_{\eta_i}^{\eta_{i-1}} n^{-s}$$

where $\eta_1 = \sqrt{\frac{T}{24}} > \eta_1 \quad \cdots \quad > \eta_\ell \overset{=\eta}{=} T^{\frac{1}{\ell+1}}$

Finally

$$u_0(s) = R(s)\psi(s)$$

Thus $\xi(s)\psi(s) = 1 + \sum_{i=0}^{L} u_i(s)$

1). choose $j_0 = 2$, $j_i = i+1$ $(1 \le i \le l)$

$j_i = i$ $(l+1 \le i \le L)$

choose first $\eta_i = T^{\frac{1}{i+3-2\sigma}}$ for $i = 1, \dots l-1$

$\xi_i = T^{\frac{1}{i+2-2\sigma}}$ for $i = l+1, \dots - L-1$,

Parts from $R \sum a_{i_0 \dots i_L} u_0^{i_0} \cdots u_L^{i_L}$

easily estimated. Critical part

$$\int_T^{2T} \sum_i |u_i(\sigma+it)|^{2j_i} \, dt$$

get with above choice

$$N(\sigma,T) = O\left(T^{\left(\frac{3}{2-\sigma}+\varepsilon\right)(1-\sigma)} \log^c T\right)$$

for any $\varepsilon > 0$ comes from u_1 & u_2 and choice of $\eta_1 = T^{\frac{1}{4-2\sigma}}$.

2) ~~In order to estimate integral over~~ $\cancel{|u_1|^{2j_1}}$

better use Phillips theory of exponent-pairs (k, l)

(4)

we have $T \le t \le 2T$

$$\sum_{N}^{2N} n^{-\sigma - it} = O(T^{k} N^{l-k-\sigma})$$

so that if $\sigma > l - k$;

$$\sum_{N_1}^{N_2} n^{-\sigma - it} = O(T^{k} N_1^{l-k-\sigma})$$

if $N_1 > T^{\frac{k+\delta}{\sigma+k-l}}$ with some $\delta > 0$

get $$\sum_{N_1}^{N_2} n^{-\sigma - it} = O(T^{-\delta}).$$

can therefore take j_1 larger and improve on above estimates. for values σ where we have a pair (k, l) for which $\frac{k}{\sigma+k-l}$ is small enough. get (with new $\eta_1 = T^{\frac{k}{\sigma+k-l}}$)

$$N(\sigma, T) = O(T^{(\lambda+\varepsilon)(1-\sigma)} \log^c T)$$

with $\lambda = \frac{3(1+2k)}{2+k-l}$; for $\sigma \geq \frac{\cancel{3}k+l}{1+2k}$

for $\sigma \geq \frac{4k+l}{1+2k}$ we have to use above estimation method also for $M_2(s)$.

(5)

Best λ gotten from Rankin's pair

$\frac{\alpha}{2}+\varepsilon, \quad \frac{\alpha}{2}+\frac{1}{2}+\varepsilon; \qquad \alpha \leq 0.329021357\ldots$

By operation ABA

$A: (k, l) \to \left(\frac{k}{2(1+k)}, \frac{1}{2}+\frac{l}{2(1+k)}\right)$

$B: (k, l) \to \left(l-\frac{1}{2}, k+\frac{1}{2}\right)$.

$p_n - p_{n-1} = O\left(p_n^{\frac{\lambda-1}{\lambda}+\varepsilon}\right) = O\left(p_n^{\theta+\varepsilon}\right)$

with $\theta = \frac{11+8\alpha}{6(3+2\alpha)} < \frac{5}{8} - \frac{4}{1027}$