$$\sum \left| \sum_{x_1}^{x_2} \frac{a_n \chi(n)}{n^{s_j}} \right|^2$$

$$f(n) = \frac{a_n}{b_n} \, \frac{\chi(n)}{n^{s_j}} \qquad x_2 \geq 2x_1$$

$$x_1 \leq n \leq x_2 \qquad \varphi_j(n) = \frac{\sqrt{b_n}}{n^{s_j - \frac{1}{2}}}$$

$$f(n) = \frac{a_n}{\sqrt{b_n}\, n^{\frac{1}{2}}}$$

$$b_n > c.$$

$$\sum \left| \sum_{y}^{2y} \frac{a_n}{n^{s_j}} \right|^2 \leq \sum_{x_1}^{x_2} \frac{a_n^2}{b_n n} \max_i \sum_{j} \left| \sum \frac{b_n}{n^{s_j + \bar{s}_i - 1}} \right|$$

$$b_n = e^{-2\frac{n}{y}} - e^{-\frac{n}{y}} \qquad \Big| \qquad \underline{\sigma_j + \sigma_i - 1 + i(t_j - t_i)}$$

$$\sum \frac{b_n}{n^{s}} = \frac{1}{2\pi i} \int \left(x_1^{w} - x_2^{w} \right) \Gamma(w)\, \zeta(s+w)$$

2

ε , $y^{-\sigma+\varepsilon}$ $H^{\frac{1}{2}-\varepsilon+\mu(\varepsilon)}$

~~use~~ we (take) largest

$$H \le y^{\frac{4\alpha-2+2\varepsilon}{1-2\varepsilon+2\mu(\frac12+\varepsilon)}}$$

$$\ll^{\varepsilon}_{T} \left(y^{2-2\alpha} + T\, y^{2-2\alpha+\frac{2-4\alpha+2\varepsilon}{1-2\varepsilon+2\mu(\frac12+\varepsilon)}} \right)$$

for small ε ; $\mu(\frac12+\varepsilon) < c\, \varepsilon^{\frac{4}{3}}$

3

$$y^{k(2-2\alpha)} = T\, y^{(k-1)(4-6\alpha)}$$

$$y^{2k-2k\alpha + 6(k-1)\alpha - 4(k-1)} = T$$

$$y^{(4k-6)\alpha + 4 - 2k} = T$$

3 & $\boxed{2}$

$$T^{\frac{k}{(2k-3)\alpha + 2 - k}(1-\alpha)}$$

$\underline{k = 3}$

$T^{2-2\alpha} z^2$

$$\cancel{z}\, x^{4 - 4\alpha} \le \frac{T z^2}{x^{4(\alpha - \frac{1}{2})}}$$

4 power

$$x^2 = \sqrt{T z} \qquad x^2 = \sqrt{T} \qquad \alpha = \frac{3}{4}$$

$\boxed{k = 3}$

$$T^{\frac{3}{3\alpha - 1}(1-\alpha)} \qquad \frac{3}{\frac{9}{4} - 1}$$

$\underline{12}$

$$\lambda(\alpha) = \min\left(\frac{3}{3\alpha - 1}, \frac{3}{2-\alpha}\right) \qquad \lambda\left(\tfrac{3}{4}\right) = \frac{12}{5}$$

4

$$\sum |R(s_j)|^3 \ll x^{-3(\sigma-1)} \int_{-2T}^{2T} |\psi_z(\tfrac{1}{2}+it)|^3 dt$$

$$\leq \int |\zeta(\tfrac{1}{2}+it)|^3 |M_z(\tfrac{1}{2}+it)|^3 dt + T$$

$$\leq \left(\int |\zeta(\tfrac{1}{2}+it)|^4 dt\right)^{\frac{3}{4}} \left(\int |M_z(\tfrac{1}{2}+it)|^{12} dt\right)^{\frac{1}{4}} + T$$

$$\leq T \log^C T \qquad \text{for } z \leq T^{\frac{1}{6}}$$

$$\sum |R(s_j)|^3 \ll \frac{T \log^C T}{x^{3(\sigma-\frac{1}{2})}} \quad \text{or}$$

$$\sum |R(s_j)|^4 \leq \frac{T^2 \log^3 T}{x^{4(\sigma-\frac{1}{2})}}$$

Thus

$$|R(s_j)| \geq \delta \quad \text{for} \ll \frac{T \log^C T}{\delta^3 x^{3(\sigma-\frac{1}{2})}}$$

points