Density theorem

Methods

Continued method

F. Carlson. Auxiliary function. \( M_j(s) = \sum_{n=1}^{\infty} \frac{f(n)^s}{n^s} \)

Jauregui formula for circle, then Littlewood's formula ending with weights depending on one side the zeros in a rectangle.

\[
\frac{2}{\pi} \int_0^\infty \frac{\sin x}{x} \, dx = \int_0^\infty \frac{f(x+it)}{x} \, dt
\]

\[
f(s) = \sum_{m=1}^{\infty} m^{-s} + \ldots + O(1)
\]

\[
f(s) \quad M_j(s) = 1 + \sum_{\pm} \frac{a_n}{n^s} + \ldots + R.
\]

essentially, either by geometric and arithmetic mean

inequality: \( V \), by length: \( u \) \( \leq \frac{1}{4} \frac{\|u\|^2}{\|u\|} \)

get integral

\[
\int_0^\infty \left| f(s) M_j(s) - 1 \right|^2 \, dt
\]

of other power, using approx

Carlson \( N(s, T) = O(T^\frac{1}{2} \log^6 T) \)

Taylor marches, approximate functional equation

\[
\lambda(s) = 4^s \quad \text{Carlson}
\]

\[
\lambda(s) \approx \frac{4}{3-2\sigma} \quad \text{Taylor marches}
\]
we have a well-spaced set of
\[ M < \frac{d}{2} \sum_{j} \mu_{j} \]

if replacing

\[ \sum_{j} \phi(s_{j}) \]

integrated over circle

\[ \frac{\pi}{a^{2}} \int_{|z - n| < \frac{a}{2}} |\phi(s_{j})|^{2} \, ds \]

could also write

\[ \phi(s) = \phi_{1}(s) + \phi_{2}(s) \]

when \( |\phi_{1}(s_{j})| + |\phi_{2}(s_{j})| \geq \frac{1}{2} \)

\[ \frac{1}{2k} \] at least

\[ M \leq 2 \sum_{v, i} \left( \sum_{j} \phi_{v}(s_{j}) \right)^{k_{v}} \]

out back advantage of higher powers easier

costants much simpler to determine

\[ M(i) \] as points \( s_{j} \) for which

\[ |\phi_{i}(s_{j})| \geq \frac{1}{2\pi} \]

\[ \phi_{i} \leq \sum_{i} M(i) \]

\[ M(i) \leq (2\pi) \sum_{\nu} \frac{k_{\nu}}{k_{i}} |\phi_{i}(s_{j})| \]
\[
\int f(z)^p \quad \int h(z)^q
\]

and later also convex theorem with \( \int f(z)^p \quad \int h(z)^q \quad 0 < a < 1 \)

\[
\sum_{x_i} \frac{x^2_i}{x^2_1} \quad a_n \quad a - 0 - i \quad a \quad a
\]

\[
\sum_{x_i} \frac{x^2_i}{x^2_1} \quad a_n \quad a^2 \quad a^2 \quad a^2 \quad a^2 \quad a^2 \quad a^2 \quad a^2 \quad a^2
\]

\[
= 0 \left( T - x^2_1 - 2a \right) + 0 \left( x^2_1 - x^2_2 \right)
\]


equation: used inequality \( \frac{1}{T} \) and \( m + \delta \)

with small \( \delta \) and either mean-value theorem

\( f(0) > f(1) \) or \( \max \: f(x) = f(0) \).

\[ \text{Results:} \quad \text{I could have obtained same by (1) using lines other than } 0 = \delta, \]

(2) using the result only set of expressions directly.

In 1945, in engineering:

\[
\log \left| -u_1 + \ldots + u_n \right| \leq k \sum a_i \ldots \ln u_1 \ldots u_n
\]

\[
+ A \sum_{k_1} \left( u_1 \right)^{k_1}
\]

Advantages: can break up \( f(x) = h(x) - 1 \)

into parts and choose each \( k_i \) (as an even integer)

such that it gives the best bound for the relevant part.

Can use a smaller \( \delta \) in relation to \( T \) (see \( \delta \))

with some arbitrary but fixed \( \delta \). Also easier to

utilize result of about exp. terms (where away fast

for which \( \sum_{m=0}^{\infty} m^{-3} = O \left( \frac{1}{m^3 - 2} \right) \); even \( \delta \).

then \( \left| M_1 (z) \right| \left| G \left( \frac{1}{2} \right) \right| \left| C \left( \frac{-\delta}{2} \right) \right| \left| k, k \geq \frac{7}{6} \right| \)
obtained results somewhat sharper than Ingham using more primitive machinery of expo... also could show that \( \pi(x) \) for \( x \) sufficiently close to \( \infty \) is \( \approx x + 0.5 \); all of Ingham's results followed although (and with as good proofs of how) \( \gamma \), and the conjecture \( \sum_{n \leq x} \frac{\ln \frac{x}{n}}{n} = O(\sqrt{x}) \)

if \( T \approx N \cdot \frac{T}{\delta} \) are small.

(holds for \( k \) an integer. \( \delta \), would imply \( \pi(x) \approx x + 0.5 \).)

Also was able to show also this instrument for detecting zeros was to look at high derivatives of \( \log \zeta(s) \) on the line \( 2 + it \)

if \( s = 1 + it \) is a zero. then \( \left( \frac{e^{it}}{s} \right) \)

could not get large.

Anything based on little words formula could not give anything better than

\[
N(i, T) = O(T^{2(1-\delta)} \log T).
\]

Namely if \( \xi(x) = 0 \) we have

\[
\xi(x) M_\xi(x) - 1 = -1 \quad \text{with means}
\]

\[
\sum \frac{x}{m^2} + R \quad \left( \text{if remainder } \leq \frac{1}{2} \right)
\]

\[
\sum \frac{x}{m^2} \text{ } \sum \frac{\text{Remainder}}{m^2} \leq \frac{1}{2}
\]

This can be used as our detecting device.
\[ \int \frac{dy}{f(y)} \leq \frac{1}{g(y) \cdot \sin \alpha} \int \frac{dy}{f(y)} \leq \frac{1}{g(y) \cdot \sin \alpha} \int \frac{1}{f'(y)} \, dy \]

\[ \alpha = 0 \text{ on boundary} \]

\[ V = \frac{\sin \alpha \cdot t}{e^t} \]

\[ \cos \left( \frac{\pi t}{2T} \right) - e^{-\frac{\pi t}{2T}} \left( \frac{\pi}{2T} \right) \]

\[ \frac{e^{-\frac{\pi t}{2T}}}{\sqrt{T}} \cdot 2 \pi \sum V(\phi) \]

\[ \sin \frac{\pi t}{2T} \cdot e^{-\frac{\pi t}{2T}} \]

\[ e^{-\frac{\pi t}{2T} \left( (\sigma - \tau) - \frac{\pi t}{2T} (\sigma - \tau) \right)} \]

\[ e^{-\frac{\pi t}{2T} \left( (\sigma - \tau) - \frac{\pi t}{2T} (\sigma - \tau) \right)} \]

\[ \sum_{n \leq 2T} \log \left| f(a + it) \right| \, dt + \sum_{n \leq 2T} \left( \frac{\pi}{4T} (\sigma - \alpha) - \frac{\pi}{4T} (\sigma - \alpha) \right) \]

\[ = \sum_{n \leq 2T} \frac{\pi}{4T} \cos \frac{\pi \alpha}{2T} \left( e^{-\frac{\pi \alpha}{4T}} - e^{-\frac{\pi \alpha}{4T}} \right) \]

\[ \sum_{n \leq 2T} (\beta - \alpha) \]

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\[ \sum_{n \leq 2T} \frac{1}{2\sqrt{2}} \int_{-2T}^{2T} \log \left| f(a + it) \right| \, dt + \frac{1}{\pi \sqrt{2}} \int_{-2T}^{2T} \left( e^{-\frac{\pi t}{4T}} - e^{-\frac{\pi t}{4T}} \right) \]
Density Heisenberg

Combining methods, approx new rep. of $\sum_{\alpha}$

Carlson, Janesics

Little woods lemma

$N(0, T) = 0(1)$

$\delta(0, T) \approx 2(\delta(0, T) \approx 2(\gamma T)$

$\delta(0) \approx 0.4$

$e^{-20}$

Titchmarsh

$1 + 20 \approx 1 + 20$

If $\delta(s) = O(\delta(s))$

$\frac{3}{2 - e^{-5}}$

(Kunitas, 1969)

Turan, Halasz 1968

$\frac{2}{\delta}$

For $|t| > 1$

$x \to \infty$

Immobility, Abel sum, Cesaro

$\frac{1}{\zeta(s)} M_{\sigma}(s) = 1 + \sum \frac{a_n}{n^s} + R(0)$

$\sum \frac{1}{n^s}$

$R(w) \frac{1}{\sin \pi \sigma} e^{-\pi \sigma x}$

$\log x \sum \frac{1}{n^s} \sum \frac{1}{n^s}$

$\int_{\delta}^{1 - 20} \int_{\delta}^{1 - \sigma} e^{-x}$

$1 - \sigma$
\[ N(T, \sigma) = Q(T^2 (1-\sigma)) \]

Terms in square sign if \( s_j = \beta_j + i \gamma_j \) real

\[ \left( \sum \frac{\gamma_j}{\gamma_j} \right) (2 + i \gamma_j) \] for large \( \gamma \)

\[ \left( \frac{\gamma}{\gamma} \right) \]

establish that one term is large in the range \( 2 \gamma \leq \gamma \leq 5 \gamma \)

\[ \phi(s_j) = \frac{1}{\epsilon(s_j - 1)} M_j \epsilon(s_j - 1) \]

\[ \phi(s_j) = 1 \] could approximate \( \phi \) by integral over \( s\)-contour

\[ \sum \phi(s_j) \]

\[ \phi(s_j) = \mu(s) + \mu(s) + \mu(s) \]

\[ \leq \frac{1}{\text{at least one } |\mu(s)| \geq \frac{1}{\epsilon}} \]

\[ \sum \]