$$L(s,\chi) = \sum \frac{\chi(\mathfrak{a})}{N(\mathfrak{a})^s} = \prod (1-\chi(\mathfrak{p})N(\mathfrak{p})^{-s})^{-1}$$

χ mod $(\mathfrak{b})$ primitive $\quad d = |\text{discriminant}|$

d. discr.

$$A = (d\,N(\mathfrak{b})\,\pi^{-n})^{\frac{1}{2}};$$

$$\varepsilon_\chi A^s\, \Gamma\!\left(\tfrac{s}{2}\right)^n L(s,\chi) = \varepsilon_\chi A^{1-s}\, \Gamma\!\left(\tfrac{1-s}{2}\right)^n L(1-s,\bar{\chi})$$

$\omega \gamma^2$

$\omega(\omega\gamma^2)^2$

$\omega^3 \gamma^4$

$\omega_1 \; (\omega^{2^{k-1}-1}\gamma^{2^{k-1}})^2$

$\omega_1 \gamma^2 (\omega\gamma)^{2^k-2}$

$L(s, \chi)$ no zeros in . . . .

Next primitive root:

$L^{\#}(s, \chi)$ $\quad \sum$

$\sum c_\chi \chi(\alpha) \neq 0$ for α primitive. $= \delta(\alpha.)$

$$c_\chi = \frac{1}{\varphi} \sum_{\alpha} \bar{\chi}(\alpha)\, \delta(\alpha)$$

$$\sum_{(\nu, p-1)=1} \bar{\chi}(\rho^\nu)$$

$$= \sum_{(\nu, p-1)=1} \bar{\chi}(\rho)^\nu \qquad \chi^\delta = 1$$

$$= \sum_{\delta \mid p-1} \mu(\delta) \sum_{\substack{\delta \mid \nu \\ 0 < \nu < p-1}} \bar{\chi}(\rho)^\nu$$

$$= \sum_{\substack{\delta \mid p-1 \\ \chi^\delta(\rho) = 1}} \mu(\delta) \frac{p-1}{\delta}$$

$\chi^2(\rho) = 1$

$\chi^\delta \equiv 1$

$\delta_0 \mid \delta \mid p-1$

$c_{\chi_0} \quad \frac{\varphi(p-1)}{\varphi(p-1)}$

$$\frac{\mu(\delta_0)\varphi\left(\frac{p-1}{\delta_0}\right)}{p-1} \delta_0$$

$\exists\ \omega_i$:

$$|\gamma\omega_i| \leq \left(\frac{4}{\pi}\right)^{\frac{r_2}{n}} \sqrt[n]{|N(\gamma)|F}$$

$|\gamma\omega_i| \geq 1$ for at least one.

$$|\gamma^{2^k}\omega_i^{2^k}| \leq \left(\frac{4}{\pi}\right)^{\frac{r_2}{n}} \sqrt[n]{N\left(\left(\frac{\gamma\omega}{\omega}\right)^{2^k}\right)F}$$

$\frac{1}{2^k}$. $|\gamma\omega_i|$ if $\gamma\omega$

$$\left|\gamma^{2^k}\omega_i^{2^k-1}\omega_j\right| \leq \left(\frac{4}{\pi}\right)^{\frac{r_2}{n}} \sqrt[n]{N\left(\left(\frac{\gamma\omega}{\omega}\right)^{2^k}\right)F}$$

if $|N(\gamma\omega)| < 1$ contradiction for Range

thus $|N(\gamma\omega)| \geq 1$.