HARISH-CHANDRA
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ROBERT P. LANGLANDS

Contents

Harish-Chandra was one of the outstanding mathematicians of his generation, an algebraist and analyst, and one of those responsible for transforming infinite-dimensional group representation theory from a modest topic on the periphery of mathematics and physics into a major field central to contemporary mathematics.

1. Kanpur and Allahabad

He was born on 11 October 1923 in Kanpur in North India. His paternal grandfather had been a senior railroad clerk in Ajmer who, to finance his son's education, had resigned his post to collect the lump sum given as severance pay, and then rejoined the railroad, his seniority lost, in a junior position. His son, Chandrakishore, later the father of Harish-Chandra, had gained admission to the highly selective Thomason Engineering College at Roorkee, which had been founded by Dalhousie in 1857, and which was responsible for the training of civil engineers for the department of public works. Every graduate was assured a position in the government services and admission was much coveted.

Harish-Chandra's father, a civil engineer, eventually rose quite high, reaching the middle echelons of the Indian Service of Engineers, and retiring as Executive Engineer of the Uttar Pradesh Irrigation Works; but his early career would have been spent in the field, usually on horseback, inspecting and maintaining the dikes of the extensive network of canals in the northern plains. Roorkee College and the effort of competing with the British on still unfamiliar technical ground seem to have produced a breed of serious-minded, conscientious men, devoted to their work and somewhat distant from their families. None the less, Chandrakishore's family did share the life of the canal posts, and Harish-Chandra,

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although not a robust child, often accompanied his father on his rounds, but it was not until later, when he was a young man and his father retired, that they became close.

In 1937, just three years before the father's retirement, the whole family was able for the first time to take an extended vacation. They travelled to Kashmir, their baggage carried by seventeen porters. Harish-Chandra, who remained a keen walker throughout his life, always recalled with pleasure the hikes in the hills with his father. Later on, after Harish-Chandra's moves to Bangalore and Cambridge, they corresponded regularly, and his respect for the high-minded, religious Chandrakishore was to be an abiding influence on Harish-Chandra.

His life as a child was divided between the canal posts and the home of his maternal grandfather in Kanpur. His mother, Satyagati Seth or, after her marriage, Chandrarani, was the daughter of a lawyer, Ram Sanehi Seth. Both he and his wife were descendants of old *zamindari* families, feudal landowners, in what is now Uttar Pradesh. One branch of the Seths appears in the 18th century as the proprietors of an important banking house who came to grief in the struggle between the East India Company and the Newab of Bengal. A more recent incident, still recounted in the family, is that it was able to offer refuge to the high-spirited but ill-fated Rani of Jhansi during the Mutiny of 1857, in which she was a central figure. As a token of gratitude she left behind a sword. Since the family is of the Rajput or Khatri caste, the men still have occasion to don it at their wedding celebrations. Harish-Chandra might have worn it as well if he had not, to the scandal of his family, insisted on a civil ceremony.

Although related to a prominent family Ram Sanehi Seth achieved his substantial social position through his own efforts, for his immediate family was of modest means, and his large house, in which Harish-Chandra was to pass much of his adolescence, was home to innumerable relatives. Precocious in his studies and often ill, Harish-Chandra did not find the turbulent atmosphere congenial, and one suspects that both home and school, where he was teased by older, rougher classmates, exacerbated an innate timidity.

However, in his grandfather's home, as in many North Indian households, music was cultivated, and Harish-Chandra took from it a love of music which he never lost, not only for the *ragas* of his homeland, but also in later life for composers of the West, above all Beethoven.

Chandrarani appears to have inherited the energy and ambition of her father and to have passed it on to her children, all of whom had distinguished careers. Satish, older than Harish by seven years, entered in 1939 the Indian Civil Service, the élite administrative corps of Imperial India, and became ultimately, after Independence, Chief Secretary of Uttar Pradesh and then Secretary to the Government of India in the Ministry of Defence and Supplies during the Indo-Pakistani War. Suresh, younger than Harish by seven years, was an engineer with the Indian Railways, and then joined the State Corporation of India. He is now executive director of a private corporation. The sole daughter, Vimala, married Jagdish Behari Tandon who, having served with the Indian Agency in Burma, where he was made a prisoner-of-war, joined the Indian Administrative Service, the successor to the Indian Civil Service, retiring as a member of the Board of Revenues of the Uttar Pradesh Government. The husband of a cousin was an admiral in the Indian Navy.

Considerable attention was given to the early education of Harish-Chandra. A tutor was hired, and there were visits from a dancing master and a music master. At the age of nine he was enrolled, younger than his schoolmates, in the seventh class. He completed

Christ Church High School at fourteen, and remained in Kanpur for intermediate college, which he finished at sixteen, and then matriculated at the University of Allahabad, where he obtained the B.Sc. in 1941 and the M.Sc. in 1943 at the age of twenty.

High-strung and frequently ill, Harish-Chandra was especially vulnerable at the time of examinations, all of which he seemed to take while suffering from some malady, serious or comic, from paratyphoid to measles. This did not prevent him from performing brilliantly. For the M.Sc., when he was examined by the physicist C.V. Raman, F.R.S., he was the first in the state of Uttar Pradesh, receiving 100% on the written test.

He had learned some mathematics, as far as the calculus, and some science from his father's textbooks but his introduction to modern science came at the university. He described many years later how Dirac's *Principles of quantum mechanics,* which he had discovered in the university library in 1940, evoked in him the desire to devote his life to theoretical physics. Two years later K.S. Krishnan, F.R.S., an excellent physicist and a widely cultivated man, was appointed Professor of Physics in Allahabad. He encouraged Harish-Chandra in every possible way, lending him books like Hermann Weyl's *Raum-Zeit-Materie* and recommending him as a research student in physics to H.J. Bhabha, F.R.S., at the Indian Institute of Science in Bangalore. The mild-mannered, gentle Krishnan inspired in Harish-Chandra not only respect but also an affection that never abated. For the boisterous, egoistical Raman and his achievements he had also, in spite of the difference in their temperaments, a high regard, but his own ascetic nature did not allow him to perceive the virtues accompanying the high-living Bhabha's extravagance.

2. Bangalore and Cambridge

The South Indian environment would have been foreign to Harish-Chandra, but he spent the first six months lodging with old friends from Allahabad, Mrs. H. Kale, who had been his French teacher at the university, and her husband Dr. G. T. Kale, a botanist who had moved to Bangalore to take up duties as librarian at the Institute.

The eager, serious student was an inviting target for the pranks of their young daughter, Lalitha, but the interruptions could not have been entirely unwelcome, for many years later, when he returned to India on a visit, she, now a strikingly beautiful young woman, became his wife. There were other interruptions. Raman, already fifty-five, had taken a liking to Harish-Chandra and would drop by unexpectedly to invite him for a walk. Harish-Chandra would also walk alone, sometimes with his sketchbook in hand, for at that time he liked to draw and to paint. He was an excellent copyist. He later gave up painting completely, although in 1951 when visa difficulties prevented him from travelling he, in his own words, made a virtue of necessity and enrolled in a painting course in the Summer School at Columbia University. A few sketches, made on vacation, remain in the family, as well as a copy of Ruben's *Le Chapeau de Paille* from the time in Allahabad, treasured by his mother-in-law, Mrs. Kale. He copied it for her as an eighteen-year-old in a gesture of affection and gratitude to a favourite teacher from a collection of reproductions of paintings from the National Gallery with which his father had presented him, choosing it as much for its French title as for any artistic reason.

Shortly before leaving Columbia, in an interview with the alumni magazine he tried to express his mathematical aesthetics in a metaphor from painting, stating that 'In mathematics there is an empty canvas before you which can be filled without reference to external reality.' In the final phrase he is thinking perhaps more of mathematics than of painting for he adds, 'The only value of mathematics lies in its internal structure.' This is an extreme view, but it has real validity if taken to refer to his own style and to express his satisfaction at having found in mathematics a subject better suited to his own inclinations than the physics he had abandoned because 'it is basically an empirical science.'

In painting, as in other things, he admired excellence. He was especially fond of the Impressionists and in his last year, often too ill to work, he spent many hours with reproductions of the paintings of Cézanne and van Gogh, reflecting on their lives, and perhaps seeing in their intensity and struggles a similarity to himself.

Gandhi's *Quit India* movement had been broken in 1942, and from then until the end of the war the independence movement was dormant. So Harish-Chandra's time in Bangalore was untroubled by politics. Indeed, although his parents had been supporters of Gandhi, his father adopting the wearing of khadi, Harish was never more than superficially touched by politics. He had strong views, which he would sometimes vehemently defend, but he was not distracted by them, and was impatient with the hypocrisy and sentimentality, perhaps simply with the welter of emotions, that politics by their very nature entail.

Harish-Chandra's career as a physicist was to be brief—two years in Bangalore with Bhabha and two years in Cambridge with P.A.M. Dirac, F.R.S. He himself does not appear to have attached much importance to the work done then, but it is of biographical interest and does occupy considerable space in his *Collected papers*.

In Bangalore there were two themes, both reflecting concerns of Bhabha and indirectly Dirac. The first, on which he wrote some papers alone and some with Bhabha, was classical point-particles, their equations of motion, and the fields associated with them. Its origins lie in a 1938 paper by Dirac in which he derived equations of motion for a classical charged point-particle moving in an external field by examining the combined effects of the external field and the field of the particle itself on a small tube surrounding the world-line of the particle. He lets the diameter of the tube go to zero, keeping only the finite part of the energy and momentum communicated to the tube, and obtains equations agreeing with those of the Lorentz theory. Similar ideas can be applied to other point-particles and the associated fields, and Bhabha and Harish-Chandra developed them extensively, especially for neutrons and their classical meson fields. This work found no echo in the literature.

The second theme, relativistic wave equations, especially for particles of higher spin, touches issues that, although somewhat peripheral, remained of concern to mathematical physicists and are still not completely resolved. It deals with problems that in the 1940s were largely algebraic and some of the papers, like those on the Dirac matrices and those on the Duffin-Kemmer matrices, are purely so. The innate algebraic facility displayed in them, and in the early Princeton papers, was transformed by experience and effort into the powerful technical skill of the papers on representation theory. As it gained in strength it lost in ease but never in resourcefulness.

Serious problems arise when attempting to construct a theory of elementary particles with higher spin. The inconsistencies that arise in attempts to include the effect of an external field appear to be the most vexing. Harish-Chandra alludes to this problem and even suggests, in an appropriately tentative introduction to one of his three papers on the topics, that his efforts might lead to its solution, but sets himself a more modest goal.

Apparently there are several desirable features for a relativistic wave equation in addition to Lorentz invariance: (i) unique rest mass; (ii) unique spin; (iii) positivity of total energy or total charge. All these requirements were met by the Dirac-Fierz-Pauli theory but at the cost of a simple Lagrangian formulation. Harish-Chandra attempted to preserve the simple Lagrangian, the unique rest mass that he took to be greater than zero, and the positivity, without which there is no quantization, but to abandon the unique spin. He was then able, among other things, to construct a formalism for elementary particles with spin that could take on both values, $\frac{3}{2}$ and $\frac{1}{2}$. However, I understand that nowadays, when there is a great variety of particles with higher spin whose existence has been experimentally discovered, the problems appear in a much different light than they did forty years ago. Either they are dealt with in the context of supersymmetry, where the inconsistencies can by a felicitous choice of coupling constants be made to disappear, or the particles are treated as composites or resonances.

The earliest papers on point-particles had been communicated by Bhabha to the *Proceedings of the Royal Society* and had gained for Harish-Chandra not only the hyphen in his name, which was first placed there by a copy editor and which he decided to retain, but also the attention of Dirac, who had been requested by Bhabha as a special favour, the wartime mail between India and England being very slow, to correct proofs.

On the basis of this work and perhaps recommendations from Bhabha as well, Harish-Chandra had been accepted by Dirac as a research student. Not long after the war in Europe had ended he set sail for England, and was on board ship when the atomic bomb fell on Hiroshima on 6 August 1945. Cambridge had still not returned to normal and was almost deserted when he arrived to take up residence in Gonville and Caius College.

In Cambridge his personal contacts with Dirac were infrequent. He attended his lectures at first but dropped out when he discovered that they were almost the same as the book. However, he did attend the weekly colloquium run by Dirac. He found that 'he was very gentle and kind and yet rather aloof and distant' and felt that 'I should not bother him too much and went to see him about once each term'.

The work on equations of particles with higher spin belongs on the whole to the Cambridge period, but his thesis proper was on a different, although closely related, topic: the classification of irreducible representations of the Lorentz group. It was proposed by Dirac and, as Harish-Chandra later remarked, was how he got started in group representations. They were to be his life.

One of the first papers on infinite-dimensional irreducible representations had been written by Dirac himself in 1944. He introduced it with the remarks:

> 'The Lorentz group is the group of linear transformations of four real variables ξ_0, ξ_1, ξ_2, ξ_3 such that $\xi_0^2 - \xi_1^2 - \xi_2^2 - \xi_3^2$ is invariant. The finite representations of the group... are all well known and are dealt with by the usual tensor analysis and its extension spinor analysis. None of them is unitary. The group has also some infinite representations which are unitary. These do not seem to have been studied much, in spite of their possible importance for physical applications.'

This is as close as one comes to the source of the theory of infinite-dimensional representations of semisimple and reductive groups, which as it turned out were to be of limited physical significance but of great mathematical import. Soon after Dirac's initial article three papers were written classifying the irreducible representations of the homogeneous Lorentz group, one by Harish-Chandra, who solved the problem posed by Dirac, another by Bargmann in the U.S.A. and a third by Gelfand-Naimark in the Soviet Union. The paper by Bargmann was the most influential of the three. He considered not only the usual

Lorentz group defined by four-dimensional space-time but also the analogous group defined by two space dimensions and one time dimension whose representations have, surprisingly, a more complex structure, containing the discrete series of square-integrable representations, which would become, after Harish-Chandra had demonstrated their importance in general, the rogue's yarn running through the subject.

Harish-Chandra's own paper suffered from a lack of the rigour appropriate to the treatment of a topic in pure mathematics. He later commented that,

> 'Soon after coming to Princeton I became aware that my work on the Lorentz group was based on somewhat shaky arguments. I had naively manipulated unbounded operators without paying any attention to their domains of definition. I once complained to Dirac about the fact that my proofs were not rigorous and he replied, "I am not interested in proofs but only in what nature does." This remark confirmed my growing conviction that I did not have the mysterious sixth sense which one needs in order to succeed in physics and I soon decided to move over to mathematics.'

In fact, it seems that he had been preparing for the move for some time. In Cambridge he attended the lectures of J. E. Littlewood and Philip Hall, discovering mathematics as he began to doubt his vocation as a physicist. In Princeton he would write one more paper on physics and then, for all professional purposes, abandon the subject entirely. Harish-Chandra's nature was unrelentingly intense, and demanded a sharp focus. His daily and yearly routines grew simpler with the years, and his temperament became more ascetic and more impatient with richness of detail or complexity of character. The best aroused admiration and respect; to the rest he was almost ruthlessly indifferent. From himself, too, he demanded the best and had, I would guess from a distance, as a young man considerable confidence in his ability to achieve it. But he would not dabble.

So once he recognized that his talents lay elsewhere his active interest in physics ceased. However, the respect he continued to have for the subject and those whom he regarded as its greatest practitioners, above all Dirac, was enormous, amounting almost to a religious awe. He never accorded so much even to those mathematicians he most admired and was most eager to emulate, certainly not to himself or his own work.

Although he was convinced that the mathematician's very mode of thought prevented him from comprehending the essence of theoretical physics, where, he felt, deep intuition and not logic prevailed, and skeptical of any mathematician who presumed to attempt to understand it, he was even more impatient with those mathematicians in whom a sympathy for theoretical physics was lacking, a failing he attributed in particular to the French school of the 1950s.

Harish-Chandra was introspective and often reflected on his own working methods. At a conference in honour of Dirac shortly before his own death he expressed his views on the role of intuition.

> 'I have often pondered over the roles of knowledge or experience, on the one hand, and imagination or intuition, on the other, in the process of discovery. I believe that there is a certain fundamental conflict between the two and knowledge, by advocating caution, tends to inhibit the flight of imagination. Therefore a certain naïveté, unburdened by conventional wisdom, can sometimes be a positive asset.'

These remarks refer to Dirac but also, and quite consciously, to himself. However, his admiration of Kodaira and Siegel expressed perhaps a clearer assessment of his own gifts. Harish-Chandra came to mathematics relatively late and, in spite of enthusiastic initial attempts, there were broad domains of mathematics that he never assimilated in any serious way, although he learned all that he needed, which was considerable. None the less, for a mathematician of his stature and ambitions his base was narrow. He knew this; it troubled him; and he was more than a little defensive. In the circles in which he often found himself he had to be. However, what saved him was not, in my view, his intuition, of which he had relatively little, either geometric or algebraic, but an analytic power and algebraic facility unsurpassed in my experience. He of course exploited the ideas of others and techniques that were at hand—they were occasionally crucial—but by and large it is not too much of an exaggeration to say that he manufactured his own tools as the need arose, and that one of the grand mathematical theories of this century has been constructed with the skills with which one leaves a course in advanced calculus.

Over the years he kept himself informed in a casual way of developments in physics through popular articles and conversation, and it gave him great pleasure that the elder of his two daughters, Premala, chose to study it in graduate school. In his last years, no longer able to work long hours, he had more time to spend with her and his younger daughter Devaki. He would often thumb through Premi's textbooks and when she was home spent many hours closeted with her in his study to discuss all that she had learned since her last visit. That it was solid-state physics, with which he was unfamiliar, and not his first passion, elementary particle physics, only heightened the interest for him.

3. Princeton, Cambridge (Massachusetts), New York

In 1947–48 Dirac was a visiting professor at the Institute for Advanced Study in Princeton and Harish-Chandra was appointed his assistant. He remained a second year on his own. In Princeton he wrote one more paper on physics, became closer to Dirac and his family, and also formed a friendship with W.E. Pauli, F.R.S., and his wife. He had already met Pauli, first in England, probably at the International Conference on Fundamental Particles and Low Temperatures held at the Cavendish Laboratory in 1946. There, to Pauli's annoyance, he had had the temerity to suggest, correctly as it turned out, in a remark at the end of Pauli's lecture that Pauli had made a mistake. Later in Zürich, while Harish-Chandra was there for the summer to study German, Pauli had taken the occasion to invite him to his home. Above all, during his stay at the Institute, he plunged into mathematics, keen, to judge from his letters of the period, to master it all, and struck up friendships with other young visitors, for example F. Mautner, G. D. Mostow and I. Segal. Some were to last a lifetime.

Curiously enough the last paper on physics [1948*b*],[1] which dealt apparently with a mathematically well-defined problem suggested by a paper by Dirac is vitiated by a topological error. Otherwise Harish-Chandra might have anticipated by thirty years results of at least some speculative interest. He considers the motion of an electron in the field of a magnetic monopole, and professes to prove there are no bound states. It turns out there is one. So far as I can see Harish-Chandra does not observe that the eigenvalue problem he is solving is not for functions but for sections of a bundle, a point that Dirac in his own manner stressed. So Harish-Chandra goes astray when separating variables, is able to

[1] Numbers given in this form refer to entries in the bibliography at the end of the text.

imitate the relativistic treatment of the hydrogen atom, and misses the novel feature of his equation.

At the same time the mathematical springs, having finally forced their way to the surface, rushed forth in a torrent that was not to abate for two decades, and indeed continued only slightly diminished until his death.

His letters of the time reveal an eager, confident, almost brash young man. In Princeton he took courses from C. Chevalley and E. Artin, but was disappointed in Hermann Weyl, F.R.S., whose personality was perhaps too elaborate for him. At the suggestion of I. Segal he read Weil's book *L'intégration dans les groupes topologiques*. He read it carefully and quickly, immediately noticing a gap in the proof of the duality theorem. In 1949–50 he spent a year at Harvard as a Jewett Fellow to study algebraic geometry with O. Zariski with whom he seems to have got along well, but his fellowship was not renewed. Perhaps he spent too much time on representation theory, but he did learn some algebraic geometry, although little appears in his papers.

One incident is worth recounting because it shows an aspect of Harish-Chandra that was afterwards suppressed. Irving Segal was teaching a course on elementary number theory at Columbia in 1953–54 from T. Nagell's text, and one day was questioned by his students about a seemingly easy problem he had assigned, namely, to show the existence for all primes p not diving $7abc$ of solutions of the congruence $ax^3 + by^3 \equiv c \pmod{p}$. He struggled with it for the rest of the hour but was stumped. He left, promising to return to the next meeting with a solution, and worked overnight on the problem, but without success. In desperation as the class approached, he consulted the many eminent specialists among his colleagues, but to no avail. He also mentioned the problem casually to Harish-Chandra, not expecting help, but the next day Harish-Chandra observed that the genus of the projective curve being one and there being at most 3 points at infinity the existence of solutions was for $p > 7$ a consequence of the Weil theorem. For $p < 7$ it can of course be verified directly. Later appeals to E. Artin and to Nagell himself yielded more elementary solutions but none simpler.

Algebraic geometry was not all he intended to learn. In a letter to Segal of 1951, in which he first describes his results on representation theory and then expresses his regrets at not being able to travel, and indicates his intention to take a course in painting during the summer, he goes on to say that he is to lecture on topological groups in the coming year and is beginning to be attracted to classical analysis, especially function theory (although he realizes it is somewhat unpopular with the modern young men) and the theory of modular functions. He describes his plans to spend most of his time on function theory and algebraic geometry, but in the next line expresses the pleasure with which he anticipates a course of Chevalley on class field theory and zeta-functions. Turning to mathematical gossip, he mentions the 'sensational new developments in France concerning homotopy groups', noting with approval that 'Cartan certainly seems to have brought in some fresh blood into topology,' and observes that 'Chevalley is still busy with his exceptional groups,' adding that 'I am sure that after Elie Cartan he is the man who knows most about them.' Finally he reports on the work of G. Racah on the invariants of the exceptional groups, with its application to the calculation of their Betti numbers, observing in a somewhat patronizing tone, 'For a man who is not a professional mathematician, he seems to be exceptionally well-informed about groups and is undoubtedly very able.' The mature Harish-Chandra was more focused, and more subdued.

In 1950 he took up a position at Columbia University in New York and remained there until 1963, although he spent several of the intervening years abroad. The academic year 1952–53 he spent in Bombay at the Tata Institute, returning also to Bangalore, where he was met at the airport by Raman. There he was married to Lalitha Kale, now Lily Harish-Chandra, who with good spirits, generous affection, patience, and all-round competence was to pamper him for thirty years.

It is not clear whether he seriously contemplated remaining in India. Bhabha seems to have made an effort to create a reasonable permanent position in Bombay, but whether Harish-Chandra felt that the proper recognition or the proper working conditions could only be obtained abroad, he did not stay and did not return again except for brief visits. He continued to concern himself with Indian mathematics and mathematicians. The strength of his influence can be measured by the number of Indians who have contributed in recent years to representation theory and related domains.

The academic year 1955–56 was spent at the Institute for Advanced Study and 1957–58 was spent in Paris on a Guggenheim Fellowship. He and Lily lived at Sceaux, where they were often visited by André Weil. Both he and Harish-Chandra were keen walkers, and would stroll in the nearby Parc de Sceaux, accompanied by Lily, who was usually ignored.

An incidental windfall from the time in Paris was a substantial increase in salary at Columbia. The dean at Columbia, unhappy with two leaves of absence so close together, expressed his displeasure in letters to Harish who was already in Paris and who, anxious to remain there, turned to Weil for counsel. Weil, for whom dean-baiting was an agreeable diversion, was glad to provide it and pointed out to Harish-Chandra that his scientific contributions of the preceding few years had been such that a large number of universities would be glad to have him, if Columbia felt it preferred to do without his services, and indeed at a considerably higher salary, for at the time Harish-Chandra's salary was on the low side. Weil urged him to communicate all this to the dean, and even dictated an appropriate letter. I am assured that it was sent, although reluctantly. The leave was certainly extended and his salary raised.

In Paris Harish-Chandra attended lectures by Weil on discrete groups and was fired with the ambition to prove the finiteness of the volume of the fundamental domain for arbitrary arithmetic subgroups of semisimple Lie groups. Returning to Columbia he studied with great attention the papers of Siegel on reduction theory, working through the proofs repeatedly until towards 1960 he found the key to the general theorem. It belonged to a line of development that had felt the hand of many of the masters of number theory, from Gauss to Siegel, and it brought Harish-Chandra great satisfaction.

In a lecture delivered in 1955 entitled *On the characters of a semisimple Lie group* [1955*b*] Harish-Chandra was already attempting to discover the properties of the distribution character of an irreducible representation, whose existence he had established in 1952. By the time of his visit to Paris he was trying to show that it was in fact given by a locally integrable function on the group. This he first announced in 1963, but at least one essential feature of the proof was already in his mind in 1958, the reduction of the theorem on the group to a similar theorem on the Lie algebra.

If ϕ is a function on an appropriate invariant neighbourhood U of 0 in the Lie algebra then

$$f_\phi(\exp X) = \left|\det\Big\{\big(\exp(\operatorname{ad} X/2) - \exp(-\operatorname{ad} X/2)\big)/\operatorname{ad} X\Big\}\right|^{-1/2} \phi(X)$$

is a function on an invariant neighbourhood of 1 in the Lie group. The dual map $T \to \tau_T$ takes invariant distributions in G to invariant distributions on U, and in order for the reduction to function it must be shown that it takes eigendistributions of the operator $\partial(p_z)$, where p_z is an invariant polynomial on the Lie algebra associated to z. For the Casimir operator Harish-Chandra knew this, but that was not sufficient evidence that it was true in general. So in Paris, contrary to custom, for he seldom considered special cases, he passed to the group SL(3), for which there is a second generator of the centre of degree 3, and by explicit calculation showed that $\partial(p_z)\tau_T = \lambda\tau_T$ if $zT = \lambda T$. This appears to have given him the necessary confidence that it was true in general.

4. The Institute for Advanced Study

In 1961–62 Harish-Chandra spent another year at the Institute for Advanced Study, which was contemplating appointing him a professor. The offer was finally made in 1963 and he returned, remaining there with only brief absences until his death.

Although his health had been undermined by overwork he had regained his vigour and was at the height of his career, about to establish the existence and the properties of the discrete series. A tall, exceedingly handsome man, already somewhat on the thin side, he was a little timid and reserved, and his intercourse was marked by a formal courtesy which did not conceal the intensity of his feelings and was often broken by a laugh or smile, although in later years he was inclined to withdraw behind it. However in 1963 his mathematical horizons were still expanding and he was, if not easy, certainly confident and gregarious, moved by those about him and willing to gossip, his comments lacking neither the appropriate malice nor the necessary insight. He enjoyed conversation on general topics and on mathematics, where his preferred style, reflecting his enthusiasm and the pace of his work, was the monologue, but there was nothing inchoate about his thought and he spoke clearly and fluently. For those who were willing to forget their own preoccupations for an hour or two it was a great pleasure to see his ideas still red-hot from the forge.

He could listen too, respond to comments and reflect on questions, but it was difficult to turn him to topics divorced from his own concerns or to make him consider a view different from his own. However, he could change his mind and was deeply attached to younger mathematicians whose work impinged on his own and who he felt had contributed decisive ideas, even when, as sometimes happened, his initial impression had been unfavourable. The influence of R. Howe, a mathematician of a cast of mind quite different from his own whose ideas were a key to harmonic analysis on p-adic groups, is patent, and was enthusiastically acknowledged. His response to my own work, perhaps because it underlined the value of his own contributions for the theory of automorphic forms, was generous in the extreme. In later years he became convinced of the importance of J. Arthur's work on the trace formula. It and the work of L. Clozel on Howe's conjectures, to which he attached great importance, were two topics that preoccupied him at his death. He did not assimilate the ideas of others easily, but if he felt they would be useful to him he was unstinting in his efforts to put them in a form he could understand.

He was inclined to brood on his own work as well. It was a sustained, cumulative effort, and he liked to formulate explicitly the ideas that guided him. The philosophy of cusp forms, which he borrowed from the analytic theory of automorphic forms, and in terms of which he eventually cast his theory of harmonic analysis on real groups, was a favourite principle. He later transferred it to p-adic groups and introduced it as well, in a short but influential paper ([1970*b*]), into the study of representations of finite Chevalley groups. He would have regarded this as an application of the Lefschetz principle, which for him meant that real groups, p-adic groups and automorphic forms (corresponding to archimedean and non-archimedean local fields and to number fields) should be placed on an equal footing, and that ideas and results from one of these three categories should transfer to the other two. The name of the principle is one of the few traces of Harish-Chandra's early reading in algebraic geometry. In his hands it led to substantial advances in harmonic analysis on p-adic groups, but it also encouraged him to ignore the arithmetical aspects of p-adic groups and automorphic forms, which appear to be richer than the analytic.

His preferred method of proof was induction, which was particularly suited to real groups, for which he was able to reduce many problems down to $\mathrm{SL}(2, \mathbf{R})$. He compared it to high finance. 'If you don't borrow enough you have cash flow problems. If you borrow too much you can't pay the interest.' Just the day before his death he maintained in a large gathering that 'In mathematics we agree that clear thinking is very important, but fuzzy thinking is just as important as clear thinking.' None the less, he himself, although he could be wrongheaded, was never fuzzy.

At the Institute for Advanced Study there are few formal duties, but Harish-Chandra loved to lecture on work in progress. Most years found him delivering a series of talks. Once or twice his enthusiasm encouraged him to precipitance. His first lectures on the discrete series in 1961 were abruptly broken off, to be resumed two years later when the hole was patched.

In 1968 he was named I.B.M. von Neumann Professor of Mathematics at the Institute. He was elected a Fellow of the Royal Society in 1973 and later of other academies. Such honours pleased him, but for a mathematician of his stature he received very few. He was considered for the Fields Medal in 1958, but a forceful member of the selection committee in whose eyes Thom was a Bourbakist was determined not to have two. So Harish-Chandra, whom he also placed on the Bourbaki camp, was set aside. Harish-Chandra would have been as astonished as we are to see himself lumped with Thom and accused of being tarred with the Bourbaki brush, but whether he would have been so amused is doubtful, for it had not been easy for him to maintain confidence in his own very different mathematical style in face of the overwhelming popular success of the French school in the 1950s.

He travelled little in later life, two short stays in Paris and a brief visit to India. Even vacations became rare. However, he did travel to the International Congress of Mathematicians in Moscow in 1966, at which he delivered one of the general lectures, and was delighted and flattered by I. M. Gelfand's hospitality.

In 1969 he had his first heart attack, and from then on his health was a serious concern. His physician prescribed regular exercise and Harish-Chandra complied, walking in the late afternoon in the streets near the Institute with long, rapid strides at a faster pace than many of the joggers. But he could not rest on his accomplishments and did not cease working. There was a competitive streak in him that he never recognized and never mastered. It did not let him rest. Sometimes he would press himself too hard, as in his attack on the spectral theory of Whittaker functions which yielded only on the second assault, the first too sanguine attempt having been unsuccessful. During the ensuing period of enforced rest a youthful almost ebullient Harish-Chandra reappeared, chatting easily about trivial matters and discoursing passionately about his favourite painters.

His heart grew worse and in 1982 he had a third attack, from which he never properly recovered. His last year was troubled by increasing frailty, the effects of medication and the knowledge that he had little time left. A conference to celebrate his 60th birthday was planned for April 1984 but he was not to live to participate in it. A similar conference in honour of Armand Borel was held in Princeton in October 1983, and was attended, the fields of the two being so close, by many friends and colleagues of Harish-Chandra. No one knows why or how, but for the week of the conference, his vigour and force reasserted themselves. Princeton's warm, clear autumn weather prevailed and between lectures at the conference, on a lawn or a terrace of the Institute, he was the centre of a lively crowd, expressing his views on a variety of topics. On Sunday 16 October, the last day of the conference he and Lily had many of the participants to their home. He was a sparkling host. In the late afternoon, after the guests had departed, he went for his customary walk, and never returned alive. His ashes were spread in Princeton and immersed in the Ganges at Allahabad.

5. Mathematical Work

Harish-Chandra's *Collected papers* were published in 1984 and contain essays by V.S. Varadarajan, N. Wallach and R. Howe that provide a comprehensive survey of Harish-Chandra's mathematical papers, describing not only his general theory and specific contributions but also the context in which they were produced. So the following description of his achievements will be brief. His papers are with few exceptions cumulative, and to some extent accretive. It appears that by the early 1950s he had already glimpsed the outlines of the theory of harmonic analysis on real semisimple groups, and in the next ten years he marched towards it with formidable determination and resourcefulness, inventing techniques and constructions as he advanced. Even after the wave of advance had crested in the discrete series and its force been partly diverted into other channels, the tenacity in the search for solutions to technical difficulties which was a characteristic of Harish-Chandra's style remained. It helps when reading his papers if one can isolate the places where severe and steady pressure has had to be applied and separate them from the stretches where experience and strength sufficed. I have attempted something of the sort but my limited familiarity with many of the papers does not permit much confidence. With time some of Harish-Chandra's arguments have been simplified and some specific results have been shown to be consequences of other general theories. I have not alluded to any of this or to subsequent developments, nor to papers he may have left in manuscript form. They have yet to be examined.

5.1. **Apprenticeship.** In a six-month period in 1948, beginning about half a year after his arrival in Princeton, Harish-Chandra wrote five papers giving new proofs or extensions of existing theorems in the theory of Lie algebras and, to some extent, groups. The influence of Chevalley on these early papers is manifest. Among other things Ado's theorem affirming the existence of faithful representations of a Lie algebra over a field of characteristic zero is proved and generalized. So is the Tannaka duality theorem, for Lie algebras and for groups.

The paper [1951*a*] written in 1950 is transitional. The first part, in which the remarkable ability to deal with the abstract semisimple Lie algebra that was a hallmark of Harish-Chandra is already highly developed, provides the first general proof of the existence of the semisimple Lie algebra attached to a Cartan matrix. He establishes it at the same time as he proves the existence by purely algebraic means of a finite-dimensional irreducible representation of the algebra with a given highest weight. This is of course the basic theorem of the subject, and had been proved before, with quite different methods, by Cartan and by Weyl. Harish-Chandra attributes some of the ideas in his construction to Chevalley.

In the remaining three sections of this long paper he strikes out on his own. He considers infinite-dimensional representations and initiates the theory of $(\mathfrak{G}, K)$-modules, but only for complex semisimple Lie algebras, showing in effect that there are only finitely many irreducible representations with a given infinitesimal character and containing a given K-type, and that a given K-type occurs only a finite number of times in a given irreducible representation. In addition he introduces for any semisimple Lie algebra the isomorphism from the centre of the universal enveloping algebra to the algebra of elements invariant under the Weyl group in the symmetric algebra of a Cartan subalgebra. It is now known as the Harish-Chandra isomorphism.

Following I. M. Gelfand and M. A. Naimark, but working with a general complex semisimple group, he introduces in the last section of the paper the unitary principal series.

5.2. **Foundations of infinite-dimensional representation theory.** These were created rapidly, so that by 1954 he had already turned to purely analytic problems: harmonic analysis and the existence of the discrete series. The main technical achievements were the existence of analytic vectors, which allows one to purge the theory of inessential functional-analytic features and thus to pass to the almost purely algebraic $(\mathfrak{G}, K)$-modules; and the subquotient theorem, from which one can deduce the existence of the distribution character as well as an integral formula for the matrix coefficients of an irreducible representation which Harish-Chandra, under the influence of the theory of automorphic forms, later called the Eisenstein integral.

5.3. **Grappling with the Plancherel formula.** The distribution character $f \to T_\omega(f)$ associated to an equivalence class of irreducible unitary representations once introduced, the Plancherel formula is a formula

$$f(1) = \int_g T_\omega(f)\, d\omega,$$

valid for smooth compactly supported functions on the group. The integration is to be taken over an explicitly described collection of inequivalent irreducible representation of the group with respect to an explicit measure $d\omega$. For complex classical groups such a formula was found by Gelfand and Naimark. To prove it one combines integration formulas on the group resulting from the circumstance that every element lies in a Borel subgroup with elementary Fourier analysis. Harish-Chandra recognized ([1951f, 1954c]) that the proof could be extended to an arbitrary complex semisimple group but not to a real semisimple group with more than one conjugacy class of Cartan subgroups.

He began to attack the problem for real groups on several fronts. He proved the Plancherel formula for SL(2) by explicit, elementary calculations [1952], using the existence of the discrete series, and understood that as far as the representations needed for the Plancherel theorem were concerned the critical point was the construction of the square-integrable representations, often called the discrete series. The notion of a square-integrable representation had also been extracted from the results of Bargmann by Godement, but what is striking is that Harish-Chandra recognized, so far as I know before bounded symmetric domains and automorphic forms became popular topics, in the work of Bargmann and that of Gelfand and Graev on $\mathrm{SL}(n, \mathbf{R})$ the technique of constructing square-integrable representations on the L^2-sections of holomorphic vector bundles. He also showed, although he expressed himself differently and the significance of the fact was not to be realized until much later when, after his proof of the existence of the full discrete series, explicit constructions were sought, that (in current terminology) only the holomorphic discrete series could be realized on cohomology groups in degree 0.

He was marshalling other techniques as well, bringing the spectral theory of ordinary differential equations and Fourier analysis to bear. He also observed that the character of an irreducible representation was an eigendistribution of the centre of the universal enveloping algebra. This allowed him to show (the argument is not difficult) that the character is, at least on the regular set, a function given by a quotient of a rather simple form ([1955*b*, 1956*c*]). There is a well-determined denominator, and a numerator which is a linear combination of elementary functions. It is the coefficients of the numerator that have to be determined in later uniqueness arguments. It is by no means clear that the character itself is a function on the singular set as well. In particular the denominator is 0 there. So one cannot say exactly when Harish-Chandra began to suspect that the quotient was everywhere locally integrable and represented the distribution, but it was certainly not long after 1955.

The deepest sequence of papers from this period is perhaps that devoted to the limit formula, which expresses the value $f(0)$ of a smooth, compactly supported function f on the Lie algebra as a limit of derivatives of its orbital integrals. One supposes that he hoped to apply the results to the Plancherel formula itself and that he was at the time unaware how important they would be for the construction of the discrete series. Here, as everywhere in the work of Harish-Chandra, there are basic identities for differential operators which result from suppressing, for various reasons, coordinates that are in some sense polar and keeping only radial coordinates. The identity that expresses the orbital integrals of zf in terms of those of f when z is an invariant differential operator with constant coefficients on the Lie algebra is, along with a similar identity on the group, the critical one in these papers and is still basic. Harish-Chandra writes it as $\phi_{\partial(p)f} = \partial(\overline{p})\phi_f$. Another technique that appears for the first time here is reduction to the semiregular elements. It was to be used over and over again. Otherwise the limit formula results from combining integration formulas with elementary techniques from Fourier analysis, and from properties of the fundamental solution of some second-order hyperbolic equations with constant coefficients.

The search for an explicit Plancherel formula is a problem in spectral theory. The formula can also be regarded as expressing a function transforming on the left and right under given irreducible representations of a maximal compact subgroup K as an integral of matrix coefficients of irreducible representations. Apparently attempting to obtain a handle on the explicit measure in the Plancherel formula Harish-Chandra considered in reference [1958*a*, 1958*b*] functions bi-invariant under K, for which he seems to have (correctly) believed that the discrete series was irrelevant, so that he had all the ingredients of their harmonic analysis at his disposal.

Here one is dealing with a higher-dimensional version of the classical spectral theory on a half-line. The elementary spherical functions which are the elements of the expansion satisfy differential equations that, when the K-invariance is taken into account, yield an overdetermined system. The central topic of the papers is the asymptotic behaviour of the spherical functions, for Harish-Chandra shows that the Plancherel measure is given, as in the classical theory, by the coefficients of the asymptotic expansion. He shows that it has an asymptotic expansion by generating a series by recursion, checking convergence and then verifying that it satisfies the necessary differential equations by interpolating between those values of the parameter that correspond to finite-dimensional representations. For the finer study of the expansion he is able to reduce the equations to a form that enables him to exploit along rays a method much like variations of parameters. The coefficient that appeared in the asymptotic expansion he labelled the c-function. He gives an integral formula for it and proves what he later referred to in a more general context as the Maass-Selberg relations. The basic ingredients of his harmonic analysis, especially the weak inequality and the Schwartz space, are implicit in these papers. All that is missing, apart of course from the discrete series, is the Bhanu-Murty-Gindikin-Karpelevich device for reducing the calculation of the c-function to the rank-one case. However, the explicit formula for rank-one groups does appear.

5.4. **The discrete series.** The basic results were announced in his papers of 1963. The first is that every invariant eigendistribution, in particular every character, is a locally summable function. The others, more difficult to state, provide the full discrete series. However, only later was he able to describe explicitly the characters of the representations of the discrete series on the elliptic set and thereby parametrize them. These are the central results in the representation theory of real semisimple groups and the proofs were long and arduous, requiring several difficult technical steps which were dealt with in a sequence of papers culminating in papers [1965c] and [1966b], now referred to as *Discrete series* I and II.

There are two forms to the theorem that an invariant eigendistribution is a function. On the Lie algebra the differential operators of which the distribution is to be an eigendistribution are the invariant constant coefficient differential operators. On the group they are the elements of the centre of the universal enveloping algebra. The theorem is first proved for the Lie algebra and then transferred to the group.

Harish-Chandra first proves, in a local form, that an invariant eigendistribution of the Casimir operator that is supported on the nilpotent elements is zero. The crux of the matter is, of course, to show that there are transverse directions on the symbol of the differential operator that there is no possibility of cancelling. The Jacobson-Morozov lemma, which was to reappear in his work on p-adic groups, is an important tool.

Local summability on the Lie algebra is proved in reference [1965a]. As already remarked it is not difficult to show that on the regular set the distribution is given by a function F. Moreover he has proved in a preceding paper (the key being the identity $\phi_{\partial(p)f} = \partial(\overline{p})f$ for orbital integrals) that F is locally summable and thus defines a distribution T_F. So he needs to show that $T - T_F = 0$. By induction on the dimension of the algebra he shows that it is supported on the nilpotent elements. For some r and some c the equation $\big(\partial(\omega) - c\big)^F T = 0$ is satisfied, $\partial(\omega)$ being the Casimir operator. By an induction on r he is allowed to assume $r = 1$. Then $\big(\partial(\omega) - c\big)(T - T_F) = T_{\partial(\omega)F} - \partial(\omega)T_F$. Since the right side of this equation is a distribution defined by a function it can be studied by integration by parts. Examining it around semiregular elements where it is shown to be 0 by induction and by reduction to $\mathrm{SL}(2, \mathbf{R})$, which can be treated directly, one shows that it is 0. Thus $\big(\partial(\omega) - c\big)(T - T_F) = 0$. Since $T - T_F$ is supported on the set of nilpotent elements it follows from the first paper of the sequence that it is 0.

In reference [1965*a*] he also proves the theorem that is critical for transferring the results from the algebra to the group. It states that an invariant differential operator that annihilates all invariant functions also annihilates all invariant distributions.

The existence and uniqueness of the invariant eigendistributions, which turn out to be the discrete series characters, is first proved on the Lie algebra. There the existence is in essence proved by writing the distributions down as an explicit Fourier transform. The existence on the group is proved by transferring in patches invariant distributions from the algebra to the group by the duals of transformations on functions of the form $f \to \phi$ with $\phi(X) = \xi(X)f(\exp X)$, ξ being a fixed function. The distributions are specified by their restriction to the open set of regular elliptic elements. Uniqueness is obtained by moving across semiregular elements to the open sets determined by the regular elements in other conjugacy classes of Cartan subgroups, using the growth conditions imposed to force most of the constants in the numerators to be 0. It involves also integration by parts and the use of the differential equations to match the constants across semiregular elements.

In 1966 he finished his study of the discrete series [1966*a*, 1966*b*], proving that the eigendistributions he had constructed are indeed characters of square-integrable representations, and turned to the harmonic analysis. The function Ξ, which defines the rate of decay demanded of functions in the Schwartz space $\mathcal{C}(G)$, appears and half of reference [1966*b*] is devoted to the basic properties of this space. To show that the eigendistributions that he has constructed and labelled Φ_λ are characters of square-integrable representations he must, first of all, verify that their Fourier coefficients with respect to a maximal compact subgroup are square integrable, or better, lie in $\mathcal{C}(G)$. The eigendistributions are tempered, in other words they lie in the dual of $\mathcal{C}(G)$, and thus by the theory of $\mathcal{C}(G)$ so are their Fourier coefficients, which therefore satisfy the weak inequality, the rate of growth permitted to K-finite functions in the dual of $\mathcal{C}(G)$. However, the differential equations satisfied by Φ_λ are passed on, although in somewhat altered form, to its Fourier coefficients, and they imply that slow growth must be rapid decay.

To show that the eigendistributions are tempered is a serious matter. The argument is convoluted and is made to rely ultimately on the maximum principle for the Laplace-Beltrami operator on G, although that could be avoided. Because one of the invariant distributions, later called the Steinberg character by Harish-Chandra, is constant on the set G_B of conjugates of regular elliptic elements, the necessary estimates can be reduced to one for $\int_K \phi_B(xk)dk$ if ϕ_B is the characteristic function of G_B, provided that the Fourier coefficient of the Steinberg character with respect to the trivial representation of K is 0. It is in fact 0 for all the eigendistributions. This is because an elementary estimate forces it to go to zero at infinity while the differential equations force it to grow.

The final major task [1966*b*] was to prove that the invariant eigendistributions he had constructed were up to sign the characters of the discrete series representations. The argument is similar to that of Weyl for compact groups and employs the orthogonality relations whose proofs are based on ideas from one of Harish-Chandra's first papers [1956*b*] on square-integrable representations.

5.5. **Harmonic analysis.** By the time he had completed the papers on the discrete series Harish-Chandra had all the techniques necessary for the development of the harmonic analysis at his disposal. However, in 1966 he gave a series of expository lectures on Eisenstein series and the analytic theory of automorphic forms, and these strongly influenced his view of harmonic analysis on a semisimple group and his presentation of it. In the lectures

[1970a, 1970c] the harmonic analysis of the space $L^2(G)$ is cast in the mould of that of $L^2(\Gamma\backslash G)$. Cusp forms appear, as do Eisenstein integrals, and cuspidal parabolic subgroups are displacing Cartan subgroups. The Schwartz space is there from before and from now on the harmonic analysis would be couched in terms of it and not of $L^2(G)$. As for ordinary Fourier analysis this permits the formulation of more precise theorems. He introduces the space $\mathcal{C}(G)$ attached to the ith associate class of parabolic subgroups and defined by means of the constant terms

$$f^P(g) = \int_N f(ng)\, dn$$

of functions in $\mathcal{C}(G)$ and verifies the direct sum decomposition

$$\mathcal{C}(G) = \bigoplus_i \mathcal{C}_i(G).$$

In addition he introduces the spaces $\mathfrak{A}(G,\tau)$, which are analogues of spaces appearing in the theory of automorphic forms, and for a function f in one of them defines, with the help of the differential equations it satisfies, the weak constant term f_P. The Eisenstein integral yields functions in the spaces $\mathfrak{A}(G,\tau)$. He is still having trouble with the analytic continuation and functional equations, which he wants to put in the form familiar from the theory of Eisenstein series. They would be dealt with in the lecture [1972] in which the Maass-Selberg relations suggested by the analytic theory of automorphic forms also occur. The Plancherel formula is proved, although the measure is not given as explicitly as in reference [1972].

The proofs of these results appeared in three long papers ([1975, 1976a, 1976b]). (The techniques of the first two papers have their origins in the two early papers on spherical functions [1958a, 1958b]). The existence and properties of the weak constant term that allow him to prove that wave packets lie in $\mathcal{C}(G)$ are proved with the help of the differential equations by variants of the method of variation of parameters. There is also a critical formula that yields the constant term of a wave packet as an integral of the weak constant term of its elements. The proof is subtle and appears, in simpler form, already in reference [1958b]. There one has a double integral

$$\int d\overline{n} \int a(\lambda)\phi_\lambda(\overline{n}h)\, d\lambda$$

to evaluate, the ϕ_λ being elementary spherical functions. Harish-Chandra replaces h by $h \exp tH$, uses the differential equations to show that this does not affect the value of the integral, and then lets t approach infinity, replacing ϕ_λ by its asymptotic expansion.

Although the Maass-Selberg relations show that all members of the weak constant term of an Eisenstein integral have the same weight on the unitary axis, in the domain where the real part of the parameter lies in the positive chamber one member dominates because of its exponent, and can be evaluated to yield a relation between the c-function and the intertwining operators. For spherical functions the argument appears in reference [1958b], and just as for spherical functions, the Bhanu-Murty-Gindikin-Karpelevich technique then reduces the calculation of the Plancherel measure to the case of maximal cuspidal parabolic subgroups.

The proof of the Plancherel formula [1976b] uses the limit formula [1957e] but perhaps not in the way suggested by paper [1970c]. Using a measure $\mu(w,v)$ defined in terms of the c-function, or the intertwining operators, he builds wave packets. Then, using the limit formula, the explicit formula for the characters of the discrete series on the elliptic elements, and the expressions for the constant terms of the wave packets in terms of the c-function, he is able to evaluate their inner products with Eisenstein integrals. After that the Plancherel formula for K-finite functions follows from formal principles, provided that one can control the growth of $\mu(w, v)$ for large values of the parameters. For this he uses an explicit expression for it that he can obtain from the relative-rank case by the product formula. For relative-rank one the alternative approach to the Plancherel formula [1970c], which exploits the limit formula [1957e], is manageable. So he applies it to evaluate a function which must be μ.

5.6. **Discrete and finite groups.** In the 1950s a large number of mathematicians began to consider the notion of an automorphic form for discrete subgroups of arbitrary semisimple groups. The domain attracted Harish-Chandra and he made two contributions. The analytic theory of automorphic forms cannot begin until one knows that the space of automorphic forms defined by a given discrete group, a given representation of a maximal compact subgroup, and a given ideal in the centre of the universal enveloping algebra, is finite-dimensional. The first general theorem of this type is proved in reference [1959a]. The argument, based on a theorem of Godement, anticipates in many respects that of the definitive results obtained a few years later, whose proof uses the general reduction theory of the paper [1962] written in collaboration with Borel. This reduction theory, which issued from the classical reduction theory and subsumes it, has been incorporated into the very foundations of the theory of automorphic forms. It yields, in particular, the finiteness of the volume of the fundamental domain for an arbitrary arithmetical subgroup of a semisimple group and the criterion, conjectured by Godement, for its compactness.

He also introduced the notion [1970b] of a cusp form into the representation theory of groups over finite fields. The subsequent theory has been erected upon the framework it provides.

5.7. **Groups over p-adic fields.** The representation theory of reductive groups over non-archimedean fields was a preoccupation of Harish-Chandra from the late 1960s. His goal was to carry the harmonic analysis of groups over $\mathbf{R}$ to groups over p-adic fields. The notes [1970d] are his first contribution to the subject, and he plunges in with the proof that an irreducible square-integrable representation is admissible and therefore possesses a character. It was perhaps the discovery of this proof that convinced him that a theory for p-adic groups parallel to the real theory could be developed. Oddly enough, in Harish-Chandra's theory, the general proof that an irreducible unitary representation is admissible was to come only at the end and only in characteristic 0, although in the meantime J. Bernstein had by other means, proved it in general.

Two results from reference [1970d], which was in some respects provisional, would be incorporated in reference [1978], whose central result is that the character of an irreducible admissible representation is given by a locally summable function. For real groups the asymptotic behaviour of the orbital integrals near the singular points is analysed with the help of differential equations. For p-adic groups the asymptotic behaviour is described by fewer elements, the Shalika germs, but they are more difficult to get a handle on. In a long chapter [1970d] Harish-Chandra succeeded in establishing, with the help of the Jacobson-Morozov lemma, important homogeneity properties of these germs, which yielded estimates from which he could establish the local summability of a function majorizing characters. The technique for estimating supercuspidal characters by expressing them as orbital integrals of matrix coefficients that has its origins in reference [1956*b*] is developed in reference [1970*b*] and appears in reference 1978 for the invariant distributions defined by cusp forms on the Lie algebra.

As for real groups the tactic [1978] is to work first on the Lie algebra and then to pass to the group. For p-adic Lie algebras there is no strict notion of invariant eigendistribution, but there is a substitute, the Fourier transforms $\overline{T}$ of invariant distributions T supported on orbits. Since the Fourier transforms of functions supported on a small compact set are uniformly locally constant one may for the local theory even allow the distribution T to have support in a thickening of the orbits. This is made precise by a finiteness theorem of Howe, which then becomes the key to the theory on the Lie algebra. The passage from the algebra to the group requires the introduction of a class of invariant distributions that is large enough to include the characters of irreducible admissible representations, and to permit localization and reduction to the centralizers of semisimple elements. Once again the key notion, that of (G, K)-admissibility, is extracted from results of Howe.

The notion appears already in his paper of [1973], which like reference [1977*b*] is a summary of results. The complete theory was expounded by A. Silberger in his *Introduction to harmonic analysis on reductive p-adic groups*. For functions, like matrix coefficients, that are K-finite on both sides, the Hecke algebra provides an adequate substitute for the differential operators. However, Harish-Chandra [1970*b*] was unable to utilize the condition of Hecke-finiteness as he had used the differential equations to study the asymptotic behaviour of functions in the space $\mathfrak{A}(G, \tau)$. The lock was turned by Jacquet with his introduction of the module now named after him and the path opened to the development [1973] of the elements of harmonic analysis: asymptotic expansions, the Schwartz space, wave packets and the Plancherel measure. It remained to prove that the wave packets exhaust the Schwartz space. For a semisimple group this amounts to two closely related theorems: the trivial representation of a given open compact subgroup K is contained in only finitely many discrete series representations, and a K-finite cusp form which satisfies the weak inequality is Hecke-finite. These are proved in reference [1977*b*], or rather deduced as consequences of another statement whose proof is not given. Harish-Chandra has stated that it was inspired by Arthur's integral formula for the character of a square-integrable representation of a real group.

6. Honours

Harish-Chandra was a Guggenheim Fellow in 1957–58 and a Sloan Fellow from 1961 to 1963. He was elected a Fellow of the Royal Society in 1973. He was elected Fellow of the Indian Academy of Sciences and of the Indian National Science Academy in 1975 and of the National Academy of Sciences of the U.S.A. in 1981. He was an Honorary Fellow of the Tata Institute of Fundamental Research, Bombay. He was awarded honorary degrees by Delhi University in 1973 and Yale University in 1981. He received the Cole Prize of the American Mathematical Society in 1954 and the Srinivasa Ramanujan Medal of the Indian National Science Academy in 1974.

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The photograph reproduced was taken at the Tata Institute in Bombay in 1973.

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