

Undated

Bill,

Since you can answer (one hopes) all questions about the unramified principal series, let me remind you of a question I asked a long time ago.

1. G - Chevalley group over non-archimedean local field
2. G° - connected component of associative group
3. $\hat{\mathfrak{G}}$ - Lie algebra of G° .

Consider pairs $\varphi = \{\hat{t}, X\}$ up to conjugacy, $\hat{t} \in G^\circ$, \hat{t} semi-simple. $X \in \hat{\mathfrak{G}}$, X unipotent. Ad $\hat{t}(X) = qX$, q is the number of elements in the residue field.

Can one associate to each such pair φ a finite non-empty set Π_φ of irreducible representations of G , which occur as constituents of the unramified principal series such that

- (i) The sets Π_φ are disjoint and exhaust the irreducible representations occurring in the principal series
- (ii) The sets Π_φ are square integrable \iff there is no proper parabolic subgroup whose Levi factor M contains t while its Lie algebra \mathcal{M} contains X . (A generalization of indecomposable representation.)
- (iii) If ξ : taking $T(F)$, the split Cartan subgroup, to \hat{L} is defined by

$$|\lambda(t)| = g^{(\text{There is perhaps a minus sign here.})\langle \lambda, \hat{\lambda} \rangle} \quad \hat{\lambda} = \xi(t)$$

and if χ is the character of $T(F)$ defined by

$$\chi(t) = \hat{\lambda}(\hat{t}) \quad \hat{\lambda} = \xi(t) \quad \varphi = \{\hat{t}, X\}$$

then each element of Π_φ is a constituent of $PS(\chi)$

- (iv) In $SL(2)$, there is such a pair $\hat{t} = \begin{pmatrix} q^{1/2} & 0 \\ 0 & q^{-1/2} \end{pmatrix}$, $X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. If one imbeds $SL(2)$ in G° as a principal three dimensional subgroup Π_φ should consist of the special representation alone.

Bob

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