July 2, 1980

Dear Bill,

I came across, almost by accident, in the PU store a book that is serving me very well as an introduction to the renormalization group. What amazes, perplexes, and even dismays me, is that in spite of the apparently tremendous difficulty of the underlying mathematical problems, these physicists get by with the crudest and most elementary of mathematics. This is either a cliché or a triumph of Thom's point of view. I was first struck by this in the chapter of Landau-Lifshutz on second order phase transformations. Heuristic discussions with the initial terms of power series expansions supported by the systematic assumption that all probability distributions are gaussian and by arguments that are at first specious, but whose real foundations elude me, allow them to reach reasonable conclusions without ever having to understand what, from a statistical or combinatorial viewpoint, is really happening.

The book is Ma's set of notes in the *Modern Theory of Critical Phenomena* and as far as I can tell the renormalization group, with which you are probably familiar from quantum field theory, can be introduced because the hamiltonian at the microscopic scale reappears at larger scales as the free energy or, better, because that hamiltonian gives the probability distribution, and can be used effectively because the probability distribution, like a Cantor set, tends to look the same at all (intermediate) scales. This is an idea which I have come across, but not understood, in expositions of the classical theories of turbulence, and also, although in a different form, in papers on dynamical systems, in particular on the Lorenz equation, and which finds some form of expression in Mandelbrojt's fractals.

To judge from his book, Mandelbrojt is as a mathematician nothing but a charlatan. He has no appetite for what most of us would regard as the mathematician's daily bread, proofs and precise statements. On the other hand, in contrast to many of us, he is eager to consider subjects which should be amenable to mathematical treatment, but have turned out to be refractory.

But the connection of the renormalization group with dynamical systems is not turning out to be what I anticipated. These Cantor-like sets appear in, for example, the Lorenz equation as attractors, and thus as a result of some global mechanism, and the problem is to develop enough technique for handling global problems, in say three variables, to be able to prove that these attractors do in fact exist and have the expected properties. However the dynamical system provided by the renormalization group is in general in infinite-dimensional space, and the problem seems to be to understand its behavior at the critical points, something entirely different and purely local.

None the less I would like to understand why the same phenomena and the same ideas turn up in such a variety of areas.

Have a good summer,
Yours
Bob

Compiled on November 12, 2024.