Undated

Bill,

Since you can answer (one hopes) all questions about the unramified principal series, let me remind you of a question I asked a long time ago.

1. G - Chevalley group over non-archimedean local field
2. $\widehat{G^0}$ - connected component of associative group
3. $\widehat{\mathfrak{G}}$ - Lie algebra of $\widehat{G^0}$.

Consider pairs $\varphi = \{\widehat{t}, X\}$ up to conjugacy, $\widehat{t} \in \widehat{G^0}$, $\widehat{t}$ semi-simple. $X \in \widehat{\mathfrak{G}}$, X unipotent. $\mathrm{Ad}\,\widehat{t}(X) = qX$, q is the number of elements in the residue field.

Can one associate to each such pair φ a finite non-empty set Π_φ of irreducible representations of G, which occur as constituents of the unramified principal series such that

(i) The sets Π_φ are disjoint and exhaust the irreducible representations occurring in the principal series

(ii) The sets Π_φ are square integrable $\iff$ there is no proper parabolic subgroup whose Levi factor M contains t while its Lie algebra $\mathcal{M}$ contains X. (A generalization of indecomposable representation.)

(iii) If ξ : taking $T(F)$, the split Cartan subgroup, to $\widehat{L}$ is defined by

$$\left|\lambda(t)\right| = g^{\text{(There is perhaps a minus sign here.)}\langle\lambda,\widehat{\lambda}\rangle} \qquad \widehat{\lambda} = \xi(t)$$

and if χ is the character of $T(F)$ defined by

$$\chi(t) = \widehat{\lambda}(\widehat{t}) \qquad \widehat{\lambda} = \xi(t) \qquad \varphi = \{\widehat{t}, X\}$$

then each element of Π_φ is a constituent of $PS(\chi)$

(iv) In SL(2), there is such a pair $\widehat{t} = \begin{pmatrix} q^{1/2} & 0 \\ 0 & q^{-1/2} \end{pmatrix}$, $X = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. If one imbeds SL(2) in $\widehat{G^0}$ as a principal three-dimensional subgroup Π_φ should consist of the special representation alone.

Bob

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