MATHEMATICAL RETROSPECTIONS

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Although there are at present many occupations that require a good deal of skill and training in advanced mathematics, mathematics itself is still often regarded as a curious profession demanding singular talents and a singular personality. My own character—apart from a certain tolerance of solitude, even a preference for it—has always seemed to me to be quite ordinary and although, as a child, I was more apt at arithmetic calculations than my classmates, my geometric intuition was not remarkable and I was never fascinated by puzzles or intellectual games. So the question why or how I became a mathematician may not have the expected answer.

The preference for solitude was perhaps acquired during an early childhood passed with little company but that of a mother and one younger sister, later two, in a tiny settlement on the western shore of Canada. On my reaching school age, the family returned to a more populated region with a parochial school where the nuns, encouraging a clear aptitude for reading and arithmetic, had me skip a class. As a consequence, the war ending and my father seeking a different kind of employment, I found myself, after another move, not quite ten years old, in another town, neither rural nor urban—in retrospect a kind of catch-all for families for whom, for one reason or another, life elsewhere had become too difficult. My new classmates, boys and girls, were all older than I, many several years older and with no academic future, just two or three years of intermittent truancy until they—at least the boys, it may be better not to think of the girls—could go off and work in the bush as loggers, returning home from time to time with money and leisure to squander. I was still too young for that. So I stayed put, passing my adolescence in a milieu outside the home for which I was never quite mature enough, and, at home, struggling to free myself, never successfully, from a family trammeled by an unfortunate mixture of Irish-Catholic piety and vestiges of English Methodism. Even so, it was an adolescence that I remember with tenderness, both the town on the sea and many of the people in it, one of whom ultimately came away with me. Curiously enough adolescence did not last long.

It had been saved from being completely idle not by school, to which I remained indifferent almost until the end but—thanks to my father's small business—by labor, the carting of building supplies—cement, bricks, lumber, shingles, everything needed for the construction of modest homes—from the truck to the storage shed, back to the truck, and then to the building site. I started at twelve or so, after school, on Saturday and in the summer, and continued, at least in the summer, until at the age of twenty I became a graduate student, with a different source of income. There were no mechanical aids at the time; everything, no matter what its weight, was carried by hand. The many days in the sun may have damaged my skin, but the many hours of physical effort as a youth also meant that my body, never frail but also initially not particularly strong, has lasted much longer than a sedentary occupation might have otherwise permitted. Above all, work and solitude, the two conditions of a mathematician's best hours, became at an early age my frequent companions.

When I reached the age at which school was no longer compulsory, I was only a few months from graduation. I was persuaded to abandon any previous plans I may have had for dropping out, the obvious possibility for adventure, and to continue to the end. The decision to do so was easier than I might have thought, perhaps because my restlessness was little more than a symptom of puberty. After very little encouragement by a generous teacher, I decided to go to university. There was a unique choice and admission was not doubtful. I applied myself to my studies for the first time, wrote examinations to compete for a small scholarship, and found myself several months later, not yet seventeen at university, and far enough from home to have all the independence and freedom I needed. I turned out to be more bookish and more timid than I had expected.

The decision to attend university had been accompanied by a rediscovered taste for reading, and a collection of brief biographies of those thinkers regarded as decisive by some of the smaller socialist parties of the thirties came into my hands. I had borrowed it from my future father-in-law, who had learned to read, although never with a great deal of confidence, at a relatively advanced age when unemployed during the Depression in classes that were a combination of adult education and political recruitment. Unfortunately it was not in that part of the small library he acquired at the time that later passed into my wife's hands. There were about a dozen biographies, for example of Einstein, Freud, Marx, Darwin, Hutton, but I do not remember all the names. Certainly, I was always tainted by a certain ambition and desire to be important that could attach itself to no particular ability or skill, perhaps because I was always younger and less mature, and therefore less capable in almost every respect, than my classmates.

However some ability at basic arithmetic and logic I certainly had, and I quickly discovered a genuine passion for speculative thinking of all kinds. These biographies of savants and scientists perhaps revealed an unfamiliar, unexpected possibility to me. So when a choice had to be made I immediately dismissed all professions, engineering or accounting, oriented, or so it seemed to me, toward the practical or the pecuniary, and dull in comparison with mathematics or the even more glamorous physics. I opted for the adventure of these two. Lack of experience meant that the attractions of other natural sciences like chemistry or biology were unknown to me. A genuine innocence of basic English grammar and of history veiled any talent or curiosity that might have suggested even a moderately successful academic career outside the pure sciences.

It turned out over time that I had a severe disability as a physicist. I was certainly fascinated by mathematical explanations of natural phenomena, examined their logic with great care, more care in fact than my professors could appreciate, but did not have the right eye for the phenomena themselves, so that left on my own, where I preferred to be, my focus was off. It might have been corrected by a little guidance; perhaps not. In mathematics it was also frequently off, but never, I believe, irretrievably.

In comparison with today's pace and prerequisites the syllabus was leisurely, but I had in every respect a lot of neglect to remedy, not alone in mathematics. The early fifties were still a post-war time. Not only had Europe appeared to recover, if only unfortunately for a brief time, its former intellectual splendor, but there were in Western Canada a good number of refugees, some of my age, whose education had been far superior to mine, and with whose attainments mine compared very unfavorably. So I was in a hurry and on my own. It was, fortunately, also a time when, in mathematics, there was a considerable

residue of deference to the Continent, even to, indeed especially to Russia and, in literature, to the British classics.

Although, Beowulf aside, the British classics are usually considered to begin with Chaucer or even Shakespeare, they were, in various ways, redolent of a more distant past and of the present greatness not only of England but of those lands to which its culture was heir, from Homer or Socrates to Rutherford or Yeats. Whatever misgivings I acquired over the years about the British Empire and the Western past that I saw then from the fringes, where there were no ancient monuments, where recent events were often best forgotten, and the present insipid, my first views of them were a revelation of previously unknown intellectual possibilities. I did not then see the wanton destruction and malevolence with which the glory, even the purely intellectual glory, was riddled.

Although I had, for reasons I can no longer recall, briefly attempted at the age of eight to learn French from the "Books of knowledge" bought by my mother from a door-to-door salesman to encourage me, I very soon gave up, so that apart from some compulsory hours in high-school that left little impression on me, I had no further acquaintance with this or any other language until arriving at university. There, on being informed, perhaps correctly at the time, that a knowledge of French, German and Russian, even Italian, were essential to the education of any self-respecting mathematician, I began to study the first three with great eagerness but initially little success, and certainly with no experience of the living languages. That came much, much later, as did any professional use of them. Russian was perhaps the one language of the three that I came to appreciate the most. Although I came to read it, write it, and understand the spoken word with some facility, it is unfortunately the only one of the three that I never learned to speak and remains one of the unrequited intellectual loves of my life. I also regret having had no introduction to Latin or Greek. There may still be time! Latin, especially, seems to me of tremendous value to a mathematician inspired by the historical continuity and modern richness of his subject, but a friend argues cogently that to understand the importance of the Greek tradition to and in mathematics and, in some sense, to understand the very essence of mathematics, Ancient Greek counts for much more.

I am grateful, on the other hand, that I undertook to become a mathematician in an era when it was still a window, both geographical and historical, on the cultural world as a whole and when travel abroad as a mathematician was still an occasion to escape, if only temporarily, from North American insularity. Although English was widely used even when I began in my thirties to travel, it was not yet universal, so that with some diligence and just a little rudeness even a rudimentary knowledge of a European language could be brought up to snuff reasonably quickly, and a genuine familiarity with the culture could begin. Then and now, but now is more difficult, the art to be learned for a native speaker of English is either how to deal with those who wish to interpose constantly, without questioning its value, necessity, or, above all, its courtesy, whatever familiarity they have with his language or how to avoid them. I prefer the second. For me, it makes all the difference between, on the one hand, a voyage or a stay that offers the additional soul promised by Jacob Burckhardt[1] and, on the other hand, a brief stint in hell, trapped in the preconceptions of others. In fact, of course, a genuinely polyglot world and the variety it

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[1] *Noch eine Sprache noch eine Seele* but I cannot remember where I saw this phrase, perhaps in his correspondence.

promotes continue to disappear. To act as though this were not so is just a crotchet that invites misunderstandings and vexations.

Europe was, luckily, only a beginning for me, although it did not come first. It took, nevertheless, a long European apprenticeship, before I again ventured beyond it. There is much in the rest of the world in the way of cultures and languages from which there is a great deal of profit and pleasure to be had. They are certainly neglected and are at best struggling to survive, but they still offer many opportunities for a mathematician to express himself without abandoning either his scientific or his scholarly goals. Despite all, there remain many men and women who respond with a humbling generosity to efforts to participate in their culture or to profit from their literature, willing to share their past, their present and themselves.

During the first five years, as an undergraduate—once I had remedied, on my own or following courses, other deficiencies—and when taking the first post-graduate degree, I was largely preoccupied with acquiring basic mathematical skills. The summers were taken up with gainful employment and courting. So there was almost no time for mathematical reflection. It was only when I went to Yale as a doctoral student that I began to think uninterruptedly—and with no pressure—about mathematics. It was a marvelous two years. There were influences that were, at the time and later, tremendously fruitful. Although I did not reach maturity at Yale, not until about seven years later, I occasionally ask myself—a fruitless question of interest to no one but me—whether I might have been a better mathematician if I had reached the age of seventeen with more mathematical experience and had spent the five years from seventeen to twenty-two with four months a year to myself with no lumber and no cement, but with a few books. Perhaps not! Much is a matter of luck!

Mathematical maturity is anyhow an uncertain concept, for the mind's natural competence seems to change with age, its purview variable. Certainly in the first few years after Yale, I harbored various ambitions, some acquiring more precision than others, and in accordance with these ambitions, largely unrealized, took, in the way of the young, as models for emulation three mathematicians. Two of the names will be surprising to colleagues, but only in relation to me. Indeed, all three, Harish-Chandra, Alexander Grothendieck, and A. N. Kolmogorov were quite beyond me in power. Harish-Chandra's influence on me will be clear to many. With Grothendieck and Kolmogorov it is more a respect for their goals than a full understanding of what they achieved.

Both Harish-Chandra and Grothendieck were engaged in constructing theories. They had in common a trait that, oddly enough, is very rare among mathematicians but that commands an unconditional respect. Not satisfied with partial insights and partial solutions, they insisted—not so much in the form of intentions or exhortations as in what they brought to pass—on methods that were adequate to establishing the theories envisaged in their full natural generality. Harish-Chandra's supreme technical power revealed itself in a novel field, infinite-dimensional representations, whose implications are still not entirely clear, certainly not fully accepted. I myself found my way to it early, and for a time, until I accepted my own limitations, was persuaded that any worthwhile mathematics had to be on the level at which he functioned. Grothendieck, in contrast, influenced, indeed entirely reshaped, a more mature field, algebraic geometry, one that had seen almost two hundred years of development by some very great mathematicians, even three hundred years if one begins, as one well might, with Descartes. I could admire the quality he shared with Harish-Chandra

and even incorporate some of his constructions in my own reflexions, but it is only slowly over the years, as my mathematical activity acquires a more reflective, more historical tinge, that I am coming to appreciate the extent and depth of his reformulation of geometry.

Kolmogorov and his mathematical style remained largely a dream. Even towards the end, I have little real understanding of his achievements. He was not only the mathematician as savant with papers in at least five languages, but also a powerful analyst, with solutions of difficult specific problems in pure mathematics to his credit and with decisive insights into the use of mathematics as a tool in understanding the natural world. Although I abandoned the serious study of physics quite early, a fascination with the attendant mathematical concepts remained. The mathematical analysis of wave motion, as found in Rayleigh or Maxwell, had charmed me as a student, and there is a part of me that would have been pleased to spend, in some small corner of a meteorological laboratory, a lifetime with the calculation and simulation of fluid motion. There are also grand mathematical problems connected with turbulence and renormalization on which I later spent a good deal of time but, unlike Kolmogorov, with little to show for it.

What I have achieved has been largely a matter of chance. Many problems I thought about at length with no success. With other problems, there was the occasional inspiration— indeed some that astound me even today. Certainly the best times were when I was alone with the mathematics, free of ambition and of pretense, and indifferent to the world. They were more seldom than necessary, for my career—and my personal life as well, but that is another matter—has been blessed.

I was saved from my own early timorousness by Salomon Bochner who insisted, despite my misgivings, even fear, for it was a difficult subject in which I had at the time no experience and a mathematical milieu, Princeton, by which I was still overwhelmed, that I offer at the University a course in class field theory, at that time in the early sixties a subject familiar only to a very few, often condescending, specialists. The handful of students and auditors learned little, but I learned a lot. Not too much later, discouraged after what seemed to me then a barren year, I was flattered and my spirits lifted by Harish-Chandra's privately expressed praise. Thanks ultimately, I surmise, to his good opinion, I was offered a position that more than compensated whatever disadvantages I may have suffered at the beginning. In the second half of my life I have been coddled so that I have only myself and my inadequacies to blame for any failures.

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