Peter, Thank you for your papers, especially the one related to automorphic forms. I am attracted by the possibilities offered by your use of the group B.

At the risk of repeating myself and more for my own sake than for yours, I recall what seem to be those aspects of automorphic representations which, at the moment, demand the most work and promise the most. I exclude special aspects, like endoscopy or the construction of the tempered representations of p-adic groups. They are essential and difficult, but are at this point better treated as special branches of the subject that require highly focussed knowledge.

(1) Proof of functoriality over number fields with the trace formula. There is the analysis, as with Ali, and a comparison of the number and nature of two diophantine "equations": those given by counting the number of pairs (Galois group + imbedding in $^LG(\mathbf{A})$) on one hand and whatever the result of the process described in my Rogawski paper gives. This is a problem that will demand mathematical strength, mathematical competence, and a great deal of time and patience with detailed calculations of examples. It is unfortunately not work for an old man. It is, in my view, the most important issue at this stage.

(2) The geometric theory. The main issue is functoriality in the geometric context. As I have tried to explain in the Mostow lecture, this is I believe given directly by a parametrization by connections. To carry out all the proofs will demand a serious understanding of differential geometry. There are, I believe, two theories, an analytic theory with genuine eigenfunctions and a theory, favored by the Russians and others, in which the emphasis is on sheaves. The two are, again as I believe, directly related, and the correspondence between them needs to be understood. The current fascination with sheaves has led to, in my view, unfortunate emphasis on only one of these aspects.

(3) Mirror symmetry. I am not yet sure what this is. I believe that it is a duality between the theories for G and ^LG. So far as I know it is related to the ideas of Olive et al. My hope is that it is a consequence of functoriality for each of the two geometric theories, namely for G and ^LG, namely of the relation to connections, harmonic or holomorphic, and of the duality between conjugacy classes in G and ^LG, but considerable reflection is demanded, a kind of reflection that is perhaps not ridiculous for an old man.

(4) Reciprocity is a feature of the arithmetic theory. It will be hard. I have indicated what I think at present in the Qing Zou appreciation. (See the section Beyond Endoscopy on my web-site.) I would like to think about it without promising anything. I believe that mathematicians have made a mistake in giving the last of the Weil conjectures priority over the attempt by Grothendieck to construct a theory of motives. I do not know what Serre's position is! Deligne's is clear and I think misguided. I have purchased a copy of Serre's book on $N_X(p)$, referred to in your paper, to see what he has to say.

(5) Your joint paper "Families..." opens up, I believe, completely new vistas. Sato-Tate as such is not terribly exciting. It is a consequence of functoriality. That, however, the distribution of zeros of the associated L-functions is related to the groups B is an extremely attractive possibility. I have been aware for a long time that neither functoriality nor reciprocity seemed to offer, even conjecturally, any help with the Riemann hypothesis or related matters. It appears that they might, at least with related matters. I have not yet studied your paper, merely glanced at it, but I hope to examine it with more care in the coming months—just to keep myself informed.

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