PREFACE TO "LANGLANDS'S PROGRAM AND HIS MATHEMATICAL WORLD"

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I was quite flattered by the suggestion of Lizhen Ji that some of my more casual writings—more historical, more philosophic, or more accessible might be more appropriate or more flattering adjectives—be translated into Chinese and by his proposal that they be preceded by a few comments, also to be translated. Much that I might write appears already in the earlier essays and it is better not to repeat that here, but it can be supplemented and my impulse is to do so, to profit from the occasion to introduce order into scraps of memory, even at the cost of disappointing those readers who would prefer that I confine myself to mathematics. I shall try to offer some comments on mathematics as a profession, but I cannot resist introducing them with fragmentary recollections of the period when mathematics meant nothing to me.

It also occurs to me as I begin that I am writing for readers with experience and a background with which I am unfamiliar. Over the years, there are parts of the world with whose history, circumstances, language and literature I have become familiar, not just my native Canada and an immediate neighbour, the United States of America, whose border I could reach as an adolescent in 30 minutes on foot, but a number of European countries Great Britain, France, Germany and Russia, as well as two Middle Eastern countries, Turkey and, to a much lesser extent, Israel. There are, however, large swaths of the globe with which I am almost completely unfamiliar: South America, Africa, and Asia, a great region, extending from Saudi Arabia, Iran, India, the South-East Asian countries, China, Japan and Korea. Only with Australia and India do I have any concrete experience, in part because they were parts of the British Empire. I have seen India; I had even begun to learn Hindi, which I found appealing. This effort was interrupted but I still hope to return to it. With Australia my experience was at second hand; my grandmother's father was born in Tasmania, the son of a British private soldier. I first heard of his adventures from my grandmother when she was very old: a soldier's wife, his death, their children, her return with the children, those who survived, to England. Although I was then no longer a child, it was only later that I could put the scraps together to understand what she or I had to do with the people in the story.

I am, as I write these pages, trying to mitigate my ignorance of China with a pair of quite different books: *Le monde chinois* by Jacques Gernet and *Essais sur la Chine* by Simon Leys. What is clear from both, but especially from the first, is how ignorant I am. It would certainly be of great value to me to overcome this ignorance, but that will take longer than a few weeks. Although it is unlikely that any reader of this essay will be so ignorant of North America, its early history prior to the European colonization, and its history subsequent to those events, it is unlikely that he or she has any notion of the specific

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circumstances of that part of North America in which I spent my childhood and youth at the time in which my personality was being formed. They have changed utterly in the intervening years.

My stance as a mathematician, what I owe to mathematics, what, in so far as they are of concern to me, I regard as its purposes, have been conditioned by the first fifteen or sixteen years. Most readers of this note, who will be, I assume, largely Chinese, will be as ignorant of the circumstances of my early years as I am of theirs, even those who might be persuaded otherwise.

An earlier essay *Mathematical retrospections* was autobiographical, but was brief and no effort was made to explain the ambience in which as a child I found myself. Moreover my early life was passed in a region and during a historical period with which most, indeed almost all, contemporary Chinese will not be acquainted. It was the tail end of a brief frontier period between the arrival of the British and the Americans and their seizure of the territory at the end of the nineteenth century and the considerable European integration following the Second World. The large-scale Asian immigration had not yet begun.

Since my temperament as a mathematician was conditioned not only genetically but also by the circumstances of my childhood, these may very well be the aspect of my life about which a young mathematician might be most curious. I was more influenced by the independence, slight as it was, allowed by them, than aware of the injustices that may have permitted it. I begin by recalling these circumstances, apologizing first to those readers who may none the less be impatient with my impulse to recall my past and the past of my family, neither of which has much unusual to offer, and would rather I turn immediately to whatever observations I might make about mathematics and its traditions. Nevertheless, this essay is one of the few occasions I have had to reflect in print on my own genealogical background, thus on my immediate relation to history, which, unexceptional as it may be, is one element of any relation to history at large. So I shall recall it, as well as, in a rather distant way, the style of my childhood, which was, in its own manner, preparation for my mature years.

Although my initial impulse was to write an autobiographical preface, it appears as I proceed that I shy away from that. Especially as an adolescent, my impulses were bolder than my actions. They were uncertain and checked by a justified lack of confidence in my own possibilities, a reluctance to find myself in uneasy circumstances. It is more agreeable to reflect on less personal, less revealing topics.

I was born and passed the first two decades of my life in the vicinity of Vancouver, British Columbia, an area now overwhelmed by immigration from the Orient, above all, China and India. More precisely, I was born in New Westminster in 1936, spent the first few years of my childhood on the shore about seventy miles to the north, in Lang Bay close to Powell River, returned to New Westminster to begin school, then moved to White Rock, where I passed my adolescence, and then went to Vancouver, of which New Westminster is now a suburb, to the University for five years, leaving in 1958 for graduate school, never to return except for short visits. I recall first the geography of the area, then the circumstances there in nineteenth century and the beginning of the twentieth as well as the circumstances of my family and me. It is an area that has seen changes that, although peaceful have been definitive: a semi-rural, even partly rural environment, with a population that, apart from a visible, but small, indigenous component, still had close ties to the Old Country, generally meaning Great Britain and Ireland, and to Eastern Canada, has become more urban, much more cosmopolitan, and undoubtedly much more sophisticated, and apparently, much wealthier.

Canada, like its neighbour, the United States, like most countries, was built on conquest and oppression. In the area where the two countries now meet, this conquest is often referred to as a discovery, and this discovery, in so far as overland voyages are concerned is recent: Alexander Mackenzie reached the mouth of the Bella Coola River in 1793; Lewis and Clark reached the mouth of the Columbia River in 1805; Simon Fraser reached the mouth of the Fraser River in 1808. The first and last of these rivers lie in Canada, the second reaches the ocean in the USA.

My childhood and youth were passed in a very small compass, first New Westminster and elementary school, then White Rock and high school, then Vancouver and university, I recall the geography. Vancouver lies with its back to the northern bank of the Fraser River at its mouth; New Westminster lies just a little upstream. The mouth of the river itself is formed from alluvial islands and the region south of it, Surrey and Delta, as far as the border with the US, a matter of 15–20 miles is also alluvial. When I was child, it was almost largely farmland, although some of the farmers had employment elsewhere. Three of my uncles were longshoremen (stevedores) on the docks at New Westminster and two of these also had small farms in Surrey. As I recall the only livestock consisted of fowl. White Rock lay at the time in Surrey but on the coast, almost at the point where the border with the USA first reaches the sea, not far from the mouth of a small river, the Campbell. I had at its source a more distant relative, married to a cousin, perhaps, of my grandfather, with a genuine farm: cows to be milked, something I attempted there but never since, and fruit to be harvested. He was also a water diviner and the owner of the last horse and buggy in Langley, the municipality adjacent to Surrey.

Lang Bay, about half-way between Vancouver and Bella Coola, was isolated and on the shore. My first recollections are from there, where we lived in a rented summer house, with two neighbours, an elderly woman and her granddaughter. My memory is largely of sea, shore, the woods, which were boggy, the neighbour's fields, and a grazing goat. Occasionally but seldom, there were visitors from the south. When I came of school age, my mother, a Catholic of Irish descent, was eager to return to New Westminster where there was a parochial school and, I suppose, to her large family, three living sisters and six brothers. I flourished in the school, appreciated the nuns, who were often young, often pretty and gentle. The traditional costume, now largely, perhaps completely, abandoned, I regarded as normal. I learned to read quickly, had no trouble with the arithmetic, and skipped a grade. I liked to read, even, under the influence of the Books of Knowledge, popular at the time and pedalled door-to-door, tried, for reasons otherwise forgotten, to learn French on my own, accompanying in the various volumes a little British family, complete with dog, on its voyage to Paris and France. Like today's more desperate voyagers, I never got beyond Calais, although I was going in the opposite direction. It was many years before I returned to the language, but then with somewhat more success. My faith was also fervent for a brief period—I even toyed at the age of seven or eight with the notion of becoming a priest, something that would have corresponded to my mother's ambitions for my recently discovered academic ability, as it would have to those of many Catholic mothers of the period, but already before leaving New Westminster my faith was failing and my desire for a greater freedom growing. In particular, I wanted to leave the parochial school for the public school.

We moved to White Rock very shortly after the war, where I spent my adolescence, arriving in 1946 and leaving for university in 1953. These were not academic years. There were very few Catholics in White Rock, indeed the children with whom I consorted never saw the inside of a church. My mother was, unfortunately attached to the Church and eager, even desperate that I remain in it. The marriage was a mixed marriage and my father, to compensate for his own sins—principally, perhaps only, gambling—would join in her efforts to bring me to church every Sunday morning, where I was a reluctant altar boy to a disagreeable priest of Irish descent. He himself stayed at home. Church meant also confession, a vicious practice, to which, for me, the only response after the age of twelve was prevarication or invention. There are, of course, many other practices that are very much worse and not far to seek. I was ashamed, as an adolescent, of my inability to resist my parents' pressure and still do not like to recall my youthful weakness. Luckily, once I left home to attend University and I was then not yet seventeen, I could abandon the Church and churches completely except for one midnight mass, the occasional funeral, and some touristic visits. Also as a concession to my wife, who was pleasing her mother, who herself appears to have been negligent about such niceties, we were married in a church, not however a Catholic Church.

The Catholic Church aside, I would say that my childhood—in a society that had not yet ceased to be a frontier society, thus a society of people who had acquired independence at more of a cost to others, in this instance largely to the indigenous inhabitants, than to themselves, among people who, by and large, were indifferent to any but an extremely modest success, financial or otherwise, major success being beyond their imagination, and only a few of whom were unlucky enough to have occasion to be confronted with authority,—encouraged a natural, even if not necessarily bold, independence, a very useful characteristic for a mathematician.

I do not entirely understand my mother's relation to the Church. A good part of her large family did not take the Church nearly so seriously as she did. Her childhood had some difficult elements. Her father was, I believe, a fireman on the Canadian National Railroad, who barely survived a head-on clash with another train and did not recover from the incident, suffering in following years from epilepsy, severe psychic disorders, as well as, I understand, alcoholism. He spent a good deal of his time in an institution, although he could, apparently, return home on weekends, from Essondale to New Westminster on foot accompanied by his dog. That was a substantial trek. So my grandmother, who had been married at sixteen, was responsible for the family. She worked as a charwoman, in the houses of women who were better off. As a result, my mother, who was apparently a lively, popular young woman, on her school's basketball team, found herself wearing the cast-off clothes of her classmates. She never forgot it. My grandmother, Emily, whose maiden name was Dickson, was, so far as I know, above such feelings. She was a tremendously warm woman, beloved of her children and grandchildren. I knew her only for a few years. Dickson is not an Irish name, but my mother's ancestors seem otherwise to have been almost entirely Irish and they seem largely, perhaps entirely, to have left Ireland, usually south-east Ireland, for example, Kilkenny Co., before the famine. I believe that the South-East was not strongly affected by it. My grandfather's family name, Phelan, is distinctly Irish and has, I believe, its origins in a region farther to the west, in the town of Cork, but as I recall reading once in a history of Ireland, the tribe of the Phelans was displaced by the Norman invaders, whom it was attempting to resist, to south-eastern Ireland in the 12th Century, whence some of them came much later to Canada.

One exception to the Irish descent of my mother is an ancestor,[1] a young German named Schildknecht who left Wittenberg or Wittenberge in Germany just before the American Revolution, in which he fought as a corporal in the South Carolina Loyalist Regiment. As compensation after Great Britain’s loss, he was granted land in Ship Harbour, Nova Scotia where he settled with his wife, born in the American colonies but clearly the daughter of German emigrants. The name Schildknecht became Shellnutt and their descendants mixed with the Irish immigrants. A daughter Mary Catherine Shellnutt married an O’Bryan. My grandmother’s mother was her granddaughter. They must have had a number of male children as well because the surname Shellnutt seems to be fairly common in Nova Scotia. As I observed, three of my mother’s brothers were longshoreman. So was her paternal grandfather. He was killed in the famous Halifax explosion of 1917 when a French cargo ship that was carrying munitions exploded in the harbour leaving 2000 residents of the city dead and 9000 injured. He was not working at the time, rather he was, with his wife, on the way home from mass.

In these peripheral ways my mother’s family was affected by the fortuities of the world’s affairs. They themselves were not much concerned with these. Even the genealogical information on both sides is not traditional, but has been, by and large, the result of efforts of a later generation. My father’s family were more recent immigrants. My mother’s parents moved across Canada from Halifax to New Westminster, stopping for an unsuccessful attempt at farming in the province of Saskatchewan where my mother was born. My father seems to have been conceived in England and his mother, who was apparently not prepared for life in a tent in British Columbia, returned with her children to England for a couple of years not long after his birth. That he had two sisters, one his twin, the other born a very short time before him, did not make her life any easier.

She was the sixth child in a family of seven. Her paternal grandfather—I know nothing of her mother—had been a private in the British Army, who stationed for a while in Cork, Ireland met and married, either then or later, a woman called Mary. Nothing more is known about her antecedents, nor about her surname. She may have been Irish or, perhaps, the daughter of an English soldier. I cannot say. On the other hand, like my mother’s mother, perhaps even more so, she seems to have been a very resourceful and courageous woman. The marriage was apparently first recorded in Hobart, Tasmania, at the time their first child was born. Her husband died in Tasmania in 1845 at the age of 36, not in the course of his military duties but of an illness. His grave and gravestone are still to be found in a famous cemetery, the Isle of the Dead, in Tasmania. His wife managed not only to find her way back to England, with at least two sons, of which my great-grandfather was the youngest and two daughters, and to enrol the sons in the Duke of York’s Military School, a school near London for the orphans of soldiers. She found employment for herself in the same institution as a laundress and for her eldest daughter as well. The whole family is listed in the 1851 Census as residing in the School. The individual indications with place of birth are as follows: (i) Flowers, Elizabeth, 13, servant of Quartermaster, Enniskillin, Ireland; Flowers, Mary. F., 39, Laundress, Cork, Ireland; Richard Flowers, 10, soldier’s

[1] My information about the genealogy of my maternal and paternal ancestors, I owe to cousins, close and distant, to whom I am very grateful: Patricia Kilt, née Phelan, Sophie Josephson, and the late Beverley Erickson, née Phelan.

son, Manchester, England; Robert Flowers, 10, soldier's son, Hobart, Van Diemen's Land, Australia. The sons were not twins, but the birth of the second was only eight months after that of the first. There was also a second sister, Mary Ann, eight years old at the time. A last son seems to have died in infancy. All in all, five children were born in about five years. Apparently the older son remained in the school and then joined the British army as a private, a rank at which he remained all his life. The younger son, Robert Flowers, my grandmother's father, asked to be released into his mother's custody, became first a draper and then an auctioneer, and with time, he became, in Newcastle-on-Tyne, first a councillor and then an alderman. He seems to have been a responsible son. Although, I have no information as to his mother's fate, it is clear that his two sisters came to Newcastle-on-Tyne, presumably with him, where they married and, much later, died.

My paternal grandmother was married relatively late, as she was approaching thirty, perhaps past it. My reading of the circumstances is that she expected to remain at home, in her father's house, which I believe was a substantial house in Westgate, so far as I know a well-to-do quarter in Newcastle. However at some point her father, who was a widower, decided to marry again. His new wife was considerably younger, almost thirty years, also well-to-do, and apparently took up enough space in his home that there was no longer room for my grandmother. So she herself married, a slightly younger man, my grandfather. They appear to have been members of the same Methodist congregation, but I am not certain. So far as I know, her father's marriage, however unwelcome it may have been for her, was a blessing for him in his later years.

My grandmother was the only member of her family to emigrate. From her and from my grandfather as well, not from my mother's family, I acquired the notion of the *old country*, a notion often invoked in their house. It was represented in their home by a bust of Kitchener, labelled Kitchener of Khartoum, invoking his famous colonial exploits in Africa. I was, myself, disabused of any notion of a special relation to the old country when I later met, as a mathematician, a number of Englishmen. I may simply have been unfortunate in my first encounters. I came to know more agreeable specimens later.

Some time after my grandparents' marriage, two or three years, the business of my paternal grandfather's father, who was a cabinetmaker seems to have collapsed, whether for general economic reasons or for illness, mental or physical, I cannot say. His whole family moved to Canada: two daughters, both of whom were married to clergymen, one apparently a missionary to the Indians of Kispiox, in northern British Columbia, where my great-grandfather is buried, and two sons, both carpenters, presumably trained in their father's shop, one of whom was later killed in an accident during the construction of the Hudson Bay building in Vancouver. My grandmother appears to have been an unhappy woman, although she was, in comparison with her husband, her children, and my mother's family cultivated. She could play the piano and, when I began university and acquired some intellectual interests, it was from her that I borrowed the *The Imitation of Christ* by Thomas Kempis, a famous medieval work of devotional literature. I confess that I neglected to return it. So far as I know, she passed a good many hours with the Bible and other devotional matters, but by the time I became curious about the world, I had little occasion to talk to her and, she, in any case, was growing senile. Whatever cultivation she had, she had acquired, I should think, in her parents' home. I had many more maternal cousins than paternal, but there was a greater awareness of the value, at least commercial, of a university education among the paternal cousins, two had business degrees and became accountants, another had a degree in engineering from a prominent American university and worked for International Business Machines. His mother, my father's eldest sister, had been trained first as an elementary school teacher and, then, as a nurse, while her sister and her two brothers all left high-school early, presumably, at least in the case of the boys, to become apprentice carpenters. A friend of my grandfather with whom he had emigrated from Newcastle and to whom he remained close until my grandfather's death told me, at my grandfather's funeral, that he had reproached my grandfather for favouring the eldest daughter, but was told that the others would not profit so well as she from any more extensive education. What role my grandmother played in these decisions, I do not know.

I knew her as a frail, rather withdrawn woman, who had little support from her children as she aged. My father perhaps assumed more of the charge, than any other, but my mother did not cooperate. New Westminster was not a large town, and my mother's brothers and sisters would have formed a large and boisterous group, not all so pious as my mother. My grandmother did not approve of the marriage and did not, I believe, disguise her feelings. My father made, however, a better marriage than he deserved. I doubt that my grandmother was aware of his more serious failings, although she expressed in the Bible she gave him on his fifteenth birthday only a feeble hope that he would read it.

I had far more cousins on my mother's side and found them more congenial, at least as an adolescent. They were easier with each other and with their mother, although my father was certainly close to his twin sister. So far as I know, apart from me, only one or two of the very youngest of my maternal cousins attended university. This did not, necessarily, prevent them from prospering. I myself, as a high-school student, had no notion whatsoever of attending university. My dream was to quit school, as one said, as soon as the law permitted, namely at the age of fifteen, and to take to the road, hitchhiking to Toronto. Certainly, a large number of students in White Rock left at this age and found work, often seasonal, as loggers or as unskilled labourers of one kind or another. My mother, by temperament or as a consequence of her childhood experience and of my response to reading, writing, and arithmetic, will have had some ambitions for me, but more likely as a priest or a medical doctor. It was certainly noticed at the high school I attended, by observation or from the IQ tests to which we were all subject, that I had more than the usual aptitude for academic topics, but I myself was not impressed.

The few years spent in New Westminster were an occasion to meet a good many of my numerous cousins, a good proportion of whom were of about my age. New Westminster was a pleasant city in which to pass the first half of the 1940s. It was founded in 1858 as the capital of the Colony of British Columbia and remained larger than Vancouver, itself founded considerably later, until into the twentieth century. It was well and carefully planned, not large but with broad chestnut-lined streets, spacious boulevards and parks. It was a joy to be in. Thanks to the Second World War, the streets were during my early childhood almost entirely free of motorized vehicles. The port itself, on the river, did not intrude and the town, hardly more than a mile or two square was everywhere accessible to a child between the ages of six and nine. So as an introduction to urban life it could not have been gentler. I was told later by my mother that I had some trouble at first protecting myself from larger bellicose schoolmates, but I have no memory of that. The only childhood fisticuffs I remember were in White Rock. There were only two incidents. In the first, not provoked by me, I, in a burst of fury, pummelled my larger opponent and had to be pulled off him by the spectators; in the second, provoked by me, I had my nose broken.

Very few events from New Westminster are fixed in my memory: an older brother of some playmates ran away from home, and so far as I know, never returned. A classmate at the Catholic school, a girl my age, Maruka, Ukrainian I believe, whose parents were gardeners who later opened a prosperous nursery, was taken ill by one of the childhood diseases feared at the time and died within a few days. She was also a neighbour and we walked to school together. Although I was not particularly troubled by her death at the time, her image has stayed with me. I also remember her mother weeping as my own mother tried to console her, as well as a second attempt at consolation a couple of years later, just before the war's end, when a second neighbour received the notice of her son's death. My own family, uncles in particular, were not affected. Perhaps they all had children. One uncle, the youngest, served in the Air Force although he never went abroad, and a cousin, who had been born in the American state of Montana, went off to serve in the US Navy, returning to New Westminster some time later. His own father had spent some time in the USA, having run off, I believe, to join the American Navy towards the end of the First World War. My grandfather had brought him home from a similar earlier attempt, but yielded to his obstinacy. Both the father and the son married Americans.

I think that parts of New Westminster have suffered lately from urbanization and traffic, but I believe also that other parts retain their earlier charm. The year of its founding, 1858, was not many years after Fraser reached the mouth of the river that bears his name. I have recalled that the beginnings of Canada were often not laudable, but neither were those of many other countries, centuries, even millennia earlier. New Westminster and the general area, was founded and developed at a great cost to the indigenous residents. The history is recorded in Wikipedia.

> In 1879, the federal government allocated three reserves to the New Westminster Indian Band, including 104 acres (0.42 km^2) of the South Westminster Reserve, 22 acres (89,000 m^2) on the North Arm of the Fraser River, and 27 acres (110,000 m^2) on Poplar Island.[11] A smallpox epidemic devastated the New Westminster Band, reducing the band members from about 400 people to under 100. Many of the remaining Qayqayt were assimilated into other local reserves, such as the neighbouring Musqueam Indian Band. Their reserve on Poplar Island was turned into an Aboriginal smallpox victim quarantine area. For decades, the Poplar Island reserve was designated as belonging to "all coast tribes". In 1913 the federal government seized most of the New Westminster Band's reserve lands. In 1916 the remaining land on Poplar Island was turned over to the BC government.

These were not the only cases of dispossession in the mouth of the Fraser River. One curious memory is a trip in an automobile to Richmond, a large alluvial island parts of which were farmed before the war by Japanese. An account of their treatment can be found at

`https://www.japanesecanadianhistory.ca/Chapter5.html`

from which I cite three or four phrases:

1. *The dispossession of Japanese Canadians was an accomplishment of Ian Mackenzie, the Minister of Pensions and Health and the Member of Parliament for Vancouver*

Centre. In April 1942, Mackenzie had journeyed to the West Coast to accept the gratitude of his constituents for his role in the uprooting of British Columbia's Japanese population and to assure them that he would continue his efforts to obliterate what he called the Japanese menace. "It is my intention," he declared on 4 April 1942, "as long as I remain in public life, to see they never come back here."

2. *Japanese Canadians in B.C. in 1942 were concentrated in relatively few occupations, notably fishing and agriculture. In agriculture, Japanese Canadians dominated the berry and vegetable industries.*
3. *The jam cannery owners and the berry marketing agents feared that the removal of these berry farmers would produce a large decline in, or the failure of, the 1942 berry crop.*
4. *The most obvious solution, to anti-Japanese British Columbians, was to force the Japanese Canadian farmers to sell their farms...*
5. *The selling prices, consequently, were much lower...*

My trip at the age of about seven years was with a friend and his father, who was quite content with his new purchase, a cut-rate Japanese farm. At the age of seven I was aware of the situation, but do not recall that I had any moral judgment on the matter. Apparently, most of the Japanese who lost their farms moved to the East, but there was one family who returned and bought a farm in White Rock. One of the daughters was a friend of my wife and a bridesmaid at our wedding, later moving to Hawaii, marrying a young man of Chinese descent, and then, after Hawaii became a state of the USA, to California, where they raised their family. My wife hears from her at Christmas.

In White Rock there was a small area of land, an Indian reserve, reserved for the members of a local tribe, of which there were only a few members remaining. The tribe, like many others, had been decimated in the nineteenth century by a disease that arrived with the colonists. The few children went to the local school but did not remain long and were pretty much ignored by the other children. This seemed to me at the time a normal state of affairs. There were also a few Métis in the town, but they were not distinguished from the rest of the population. My parents' store and our home above it were across the street from the reserve, but not from its residences. The chief came occasionally to buy lumber or other building material and would chat with my mother, who was usually at the cash register. Those boys in the town who were fond of fishing and of solitude would spend time in the reserve because the river ran through it, as did the trail to the border and the adjacent American town of Blaine.

I was nine, almost ten, years old when we arrived in White Rock, sixteen when I left it for university, returning only in the summer, and nineteen when I left it for good. It was a pleasant, but a strange, place for an adolescent. There was the ocean and the shore, although there were few boats. Except for the crab fisherman, the owners of the few boats, namely small rowboats, were usually Americans with a summer cottage. There was little exchange between the children on our side of the border and those on the other. The Americans were considered richer and were recognizable, from the front or from the back, by a slight plumpness.

Before the war the town's principle function was as a resort village in proximity to New Westminster and, to a lesser extent because of the lack of bridges and tunnels, to Vancouver. After the war, it served a different function, or so it seems to me on reflection. The summer cottages afforded inexpensive lodging. So there were a good number of families with no father, with a father who appeared only infrequently, or a feckless father. The proprietor of the local hotel or of the local dance hall were in my eyes rich. There were other families and their children as well, but those children whose families allowed them, for one reason or another, more freedom appealed to me.

My wife has a copy of the year book of the high school with photos of the classes, thus for grades 7 to 12. They suggest that there were about four hundred children in the school. Their faces and names are by and large familiar, but many of them came from the surrounding rural areas and many kept pretty much to themselves returning home directly after school, so that I knew much less about them and their circumstances than about those in the town itself, where there were also those youths who had left early to find work full or part-time. Schooling was only compulsory until age 15. A hint of restiveness, a desire for independence, drew me to those who had left school or were free, for one reason or another, of parental constraints, but I was very young, not very bold, and could not entirely free myself from the interdictions imposed on me by my mother and, in her wake, my father. As a Catholic child, I believed, without any question that any sins would be observed not only by God above but by my recently deceased grandmother, whom I cherished, at his side.

It is not that I was up to the company of the children or youths whom I admired or envied. I had started school rather early and had skipped a grade, while a good number of my classmates had failed a grade, thus been kept back, and not just once but several times, so that they were substantially older than I was. Moreover they had substantially more freedom than I. They may not have been able to read or write with any ease, but I envied them, both the boys and the girls. My mother, curiously enough, because it was not shared by a good number of her brothers and sisters, had a fear of sin, in particular of books, not of course children's books, as a source of sin, that made life with her difficult. My father, whose Methodist/Wesleyan (in the diluted Canadian form of the United Church) background had not left him immune to sin, but had left him with a strong sense of propriety and of possible disapproval of the neighbours, provided no relief. So I had to struggle for whatever freedom I had. I did quickly take to foul language, although I could not use it at home. From the age of twelve to the age of fourteen, I could probably compete with the most imaginative or coarse of Indian taxi-drivers in Vancouver or of current American politicians, but between the ages of fourteen and sixteen my passion for this form of expression slowly dissipated. In those years I met, at one of the school dances, called *mixers* and introduced to encourage civilized social intercourse between the boys and the girls in the school, someone almost as timid as I but, in contrast to me, with plans for the future. This was a decisive, perhaps the decisive, event in my life.

A determining feature of those years may have been labour. The period after the war was an economically favourable period. My parents had moved to White Rock, away from New Westminster, probably at the urging of my mother, where they had founded a business—lumber and building supplies. My father provided the technical competence—he had training as a carpenter, although he never acquired his journeyman's papers—while my mother took care of the books. As would be normal at the time, perhaps today as well, my father was responsible for the collection of overdue accounts, which were frequent enough. This was, for him, not an agreeable task. For me, the fortunate aspect of the business was to provide me with an occupation to fill time that would otherwise have been idled away. Although in no way fragile, I was not a particularly strong youth, nor was I particularly athletic. I tried but I was younger than my classmates, so that I was never chosen for school athletic teams. On the other hand, in those days, kegs of nails, sacks of cement, agricultural tiles, plywood, plasterboard, and lumber of all kinds were loaded onto trucks by hand and unloaded in the same way. From the age of twelve or thirteen that was how I spent my time after school and on Saturdays. During the university years it was how I spent the summers, earning the funds to pay for the winter's food and lodging. It meant, above all, that I arrived at twenty reasonably robust, with a body that has not failed me, at least not seriously, over the next sixty years. It also meant that, without being particularly adroit physically, I could manage, although not with great skill, those household tasks associated with the building trades. However, as time went on I was ever more disinclined to undertake anything outside mathematics that demanded patience. With age, mathematics demands that quality more and more.

On reflection and I have, oddly enough, never indulged myself in reflection about these matters, after my early childhood, even during, I had little to do with my father outside these common labours. There was little disagreeable about them. Although he was occasionally impatient with my lack of dexterity, it was pleasant to work with my father. He was generous with my pay, adequate to allow me to indulge my juvenile sartorial extravagance, to go to the movies and so on, and, now and again, as a diversion, he suggested that I work as a swamper, a term used only in Canada, thus as a helper on the local light-delivery truck, which was not only more leisurely, with a good deal of time spent beside the driver watching the world go by, but entailed occasionally a trip to Vancouver or New Westminster for a load of cement, drainage tiles, or sashes and doors. This was hardly work!

The postwar years were prosperous; the business thrived, ostensibly under the hand of my father, but the determination was, I believe, my mother's, although this was not apparent to me at the time. My father had a modest taste for luxury. As the business prospered, they were soon able to construct a building to house it, with an apartment upstairs for the family. With a stone facing and large plate glass windows, it was at that time and place an imposing edifice. It now houses a restaurant. With time, my father was able to buy himself a Buick, at that time a luxury automobile, and to construct a house in the best part of town, on a cliff on the shore with a splendid view over the Strait of Juan de Fuca. Unfortunately, as my mother grew older and became ill, she was no longer able to save him from himself, and a vice—gambling—that had been present from the beginning, although hidden from the children, and a constant source of anxiety to her, took over. He, and thus they, slowly lost everything. By then, I was far away. It fell to my two sisters to do what they could, and it was considerable, to mitigate the disaster.

To return to my own development, how did it happen that rather than hitchhiking to Toronto, I went to university? It may have been the effect of my new acquaintance, but that would have been an unconscious effect, although I doubt that she would have encouraged my hitchhiking plans. She would certainly not have been willing to be a part of them. It would undoubtedly have meant that we parted ways.

There were two things. First of all, in the last year of high school, we had a new teacher, Crawford Vogler, and a new textbook, a textbook that introduced us to English literature. He was a very enthusiastic, very sympathetic teacher. I recall that he gave two or three students special assignments. I was asked to report on the novel *The Ordeal of Richard Feverel* by the well-known Victorian novelist George Meredith. It was all a little puzzling to me and any expectation on his part must have been disappointed. However, as I remarked above, I will have been given an IQ test and he will have been aware of any unusual talent.

That will have been the source of his disappointed confidence in me. None the less, he took, either then or on some other occasion, a full class period to explain to me in front of the other students that I must go to university. Going to university meant going to the University of British Columbia. There were then no other universities in the province and the possibility of going elsewhere would never have crossed my mind. I was impressed then and there by the suggestion and decided not only to take the entrance examinations but to study for them. I was successful; I even received a bursary to pay the academic fees.

Secondly, although my father had left school after nine years to become an apprentice, my wife, not at that time of course, was the child of a man with a more meagre educational background—born on Prince Edward Island, into a mixed, Franco-Irish marriage. His mother died when he was two years old and he was given into the care of a Scottish family. So his initial language was Gaelic, but apparently somewhat frail and uncomfortable with the robust sons of the family, he left home at an early age, working in the logging camps of Quebec, where he learned French, as spoken by the loggers. He must have been quite agile, since his task was sometimes to clear log-jams. This was done by inserting an explosive in the jam and then running away, from one log to another, in order to be clear of the jam before the explosion freed the logs—a slip would be fatal.

What he had not learned neither with the Scots nor in the logging camps was to read and write. His chance came during the Great Depression, when various unions or political parties undertook not only to feed the unemployed but also to educate them. He had kept some books, by Frederick Engels, Karl Kautsky, August Bebel and other socialist authors, that he undertook to read at that time, most of which we still have. He liked to cite at length various passages from these books. He had a good memory but reading was always difficult for him. I think his wife, my mother-in-law, gave him further lessons after their marriage. My own father could read well enough, although writing was another matter. I do not think I ever received more than one brief note from him, in an emergency. One book in particular, I took away from my future father-in-law's library to read, *The Story of the World's Great Thinkers* by Ernest R. Trattner with biographies of many of the world's renowned thinkers, for example, Copernicus, Hutton,... Marx, Pasteur, Freud,..., Einstein. I remember being particularly impressed by the story of Hutton and the age of the Earth. The book itself had been very popular, deservedly so, in the late thirties and early forties of the last century. So second-hand copies are still easily found.

I recalled at length various genealogical facts related to my family. I recall one or two related to my wife. They are striking. Her mother's family, like my mother's large with ten children, had emigrated not from England but from Scotland at the time of the First World War. My grandfather and his brother had returned to the Old Country as soldiers but they were sufficiently old that they never, so far as I know, saw battle. Lord Kitchener, who as I recalled was a familiar figure to me from my grandparents' dining room, was the Secretary of State for War for Great Britain at the beginning and had introduced the policy of sending brothers together to the battlefield on the principle that side-by-side they would fight better. The result was that many families lost all their sons at one stroke. My wife's grandfather's family seem to have been so affected, not her grandfather and one of his brother's who had also emigrated, who survived, although injured, as members of the Canadian army, but the four brothers who remained in Scotland and fought with the British army all perished, at least three apparently as a result of Kitchener's policy, which I believe was finally abandoned.

Part at least of her father's family had been in Canada much longer, descendants of a Basque fisherman and his Micmac wife, who are to be found in the census undertaken by the British after the conquest in 1763. My children are aware of their descent, although it is hardly apparent. Nevertheless, one of my daughters, the blonde among the four children, learned recently from her dentist that there is an aspect of her dental structure that is a sure sign of an indigenous ancestor.

Almost the first event marking the change from childhood years to university years were aptitude tests. They were followed by consultation with some member of the university's teaching staff. I was offered, given my arithmetic talent, such possibilities as accountancy although academic possibilities were also mentioned. I suppose I expressed an intention to study mathematics and physics and it was observed that in that case I might even want to take a Ph.D. degree. I listened and, my studies not having yet begun, returned to White Rock, where for some reason or other I visited the house of my future in-laws, who were in bed. I took the opportunity of asking my future father-in-law what a Ph.D. was. Somewhat surprisingly, now but not then, he knew. Sometime later, I consulted with a mathematician, Dr. Jennings, with the title for professors customary in Canadian universities, who suggested to me that as a mathematician there would be several foreign languages to be learned. I took his remark seriously, although, initially, seriously may not have entailed effectively. One year of French or some other language was a normal requirement. At the end of the first year I acquired a basic text for German grammar and reading it over the summer felt that I was adequately equipped in that direction, and in the second year moved on to a course in Russian.

In retrospect, these efforts were a little ridiculous, but I had occasion later to make more serious and more effective attempts to master these and other languages. I pity the mathematicians of today, not only the native speakers of English who have no occasion to learn the classical European languages, which offered until recently, outside mathematics and within mathematics as well, a great deal but also the European mathematicians and mathematicians from elsewhere, but especially the Europeans—the French who have all but destroyed Breton, the Germans, who have destroyed above all Yiddish but also less important languages as well, like Wendish/Sorbian—who now are assiduously rendering their own more and more difficult of access. They, with a notion that acquiring English is today still a cultural achievement, merit perhaps more contempt than pity. Some, of course, like the editors of Springer-Verlag, are in it for the money. It may be that the response as to whether mathematics should be a matter of several languages or of one will ultimately be given by the Chinese.

In my first year of university, my principle preoccupation was some mastery of English. As I have implied I was in high-school negligent and had ignored the basics not of English orthography but of English grammar. I was assiduous—to the amusement of the other students, some of them adults, and the teacher, Dr. Morrison—consulting the dictionary for every unfamiliar word in every poem and learning, in particular, to avoid comma splices.

The mathematics was new to me but by today's standards elementary, largely, as I recall, trigonometry. The second year was again relatively slow, mathematics and a course of logic, a subject about which, as a mathematician, I have tried to inform myself but always unsuccessfully. The physics course, the Russian course, and the course on English literature, from the beginning to the nineteenth century, Chaucer, Fielding, and a good number of other authors, all appealed to me. We had been introduced to Shakespeare in high-school.

The third year was more interesting. For various reasons the multi-variable calculus courses were not successful, a bad and indifferent teacher undermined the efforts of a conscientious and potentially excellent teacher, Dr. Leimanis. However Marvin Marcus, now very old but still, I believe, with us, recommended Courant's classic book on differential and integral calculus, which I studied, but occasionally, as with the inverse function theorem, in too superficial a manner. Either in the second but more likely third year, I had two other courses, each, as I recall, half-year courses. Dr. Christian gave an excellent course on algebra from a well-known book of Dickson. Once again, I did not always grasp adequately the interest of various important points, for example, the theory of the cubic equation. The textbook in the other course, on linear algebra and geometry was also a widely used American text, the names of whose authors I forget, but during the summer, at the suggestion of Christian, I read Halmos's book on vector spaces, widely used at the time. I confess that I fell in love with the abstraction of his presentation, a passion good in its way for a mathematician, but it is best not to be overwhelmed by it. A book that, in some sense, was a surer sign of my fate as a specialist of automorphic forms and the Hecke theory was a book that I found on my own, a translation *Modern Algebra and Matrix Theory* of a once familiar German text by Schreier and Sperner where the theory of elementary divisors is treated at length.

By the fourth year, I could give myself up almost completely to mathematics. Not entirely through a fault of my own, I had abandoned any intention I may have had to become a physicist. In retrospect, I did not and do not have the right kind of imagination, but the decisive event was a course in thermodynamics in the third year. This is a difficult subject, in particular the topics of heat and entropy and in response to a homework question I wrote an extravagantly long essay, which unfortunately I did not keep. Given my age—I had just turned nineteen—and my lack of a solid pre-university education, it probably was not so bad. The teacher, an English experimentalist, chose to mock it in class. That, I think, was the turning point. Certainly, given the nature of whatever talent I had, it was for the best. I did, that year, have a physics course that I much enjoyed, the optics course, above all the experiments. I was fortunate to have a partner, Alan Goodacre, in the laboratory experiments that were an important part of the course and that in his hands always yielded the expected, thus the correct, results. There was a division of labour, in which I took the easy half, the theory, and he the difficult half, the experiments. I still have occasion to meet him occasionally in Ottawa, where he was an experimentalist with the dominion laboratories for many years.

So, in the fourth year, I focussed except for a second course in Russian on mathematics. I learned or began to learn a great deal: function theory, in particular from the third volume, which I studied on my own, of the prescribed text, a translation of a German text by Konrad Knopf, the Weierstrass theory of elliptic functions; ordinary differential equations, including something about special functions and the beginnings of spectral theory, which I supplemented later in graduate school and the years immediately following, with the book of Coddington and Levinson and with M. H. Stone's book on the spectral theory of operators in Hilbert space, both excellent preparation for the general theory of Eisenstein series, which has been a major concern of my career and which led to its major achievement, what is often referred to as the Langlands programme. Galois theory was the principal topic in another course but it went by, I am afraid, more or less unremarked. I also participated in my fourth year at university, or more likely in the following year, in a seminar on commutative algebra, based on the book *Ideal Theory* of D. G. Northcott. In any case, I managed during the following year to write, on my own initiative, a master's thesis on some idea or problem that I found in it. None of the professors were familiar with the topic so that they were uncertain what to do. Out of the goodness of their hearts and, I suppose, because my performance was otherwise satisfactory they accepted it—even though I had had to confess at some point during the proceedings that I had found an important error in it—and let me move on to graduate school. That year 1957–58, between my four undergraduate years and my two graduate years at Yale, was one of the most demanding of my life. I had been married a year before, at a very young age, was teaching one undergraduate course, my first experience of lecturing, was taking enough courses to acquire the necessary credits for a master's degree, without which I could not move on to the important stage of a doctoral degree, and finally I was writing the master's thesis. I remember almost nothing from that year: life in a trailer with my wife; a charming girl in the freshman class I was teaching who seemed to be taken with me, an infatuation to which unmarried and otherwise uncommitted I would have been happy to respond; as well as an incident with a second professor of physics. This I remember clearly.

The occasion was the final examination of a course on mathematical methods in physics, a course for graduate students offered by this professor, an immigrant from Europe. The focus was the representation theory of finite groups, characters, orthogonality and so on. The single problem assigned was to analyze the representation of the tetrahedral group on the sum of the four tangent spaces at the vertices, each a three-dimensional space, which he thought of as provided with the natural metric. He expected the students to decompose the representation using the orthogonality relations. The best, most direct solution is of course to use a non-orthogonal basis directed away from the vertices along the edges. Then the problem is solved by inspection. He looked at all the zeros and ones in the calculation and was persuaded that I, as a thick-headed student, had inappropriately introduced the regular representation and was about to fail me, which would have meant no master's degree and no move to Yale. He seemed to be obdurate, but for unexplained reasons ultimately gave me a passing grade. Perhaps a colleague explained the solution to him.

All in all, the years at the University of British Columbia were very profitable. I learned something: how to write English, a beginning in three other languages, a little bit of physics that did no harm and, both in the courses and on my own, a good deal of mathematics. The campus too was a pleasure, small and forested. Photographs I have seen suggest that it is now an asphalt desert, but so are many places.

I add as well that I was by and large satisfied with my independent reading in various mathematical domains. It still seems to me that a mathematician's obligation, as much, even more, than proving theorems, an activity that sometimes demands more ingenuity than insight or foresight, is the preservation of the creations of the past, not necessarily all, but certainly those that sustain the subject's depth and intellectual pertinence. Although I had an exaggerated fondness for abstraction, one nourished by Halmos, it was sated by the book of Dixmier, *Les algèbres d'opérateurs dans l'espace hilbertien* which I believe I managed to read from beginning to end as a part of my studies for the master's degree. Nor was I able to maintain a sustained interest in logic, even the introductory text *The elements of mathematical logic* by Paul Rosenbloom defeated me.

I set off for Yale University full of hope and my wife followed in a couple of months with our first child. A second one was to be born less than a year later. At Yale I followed two or three helpful courses, one on the basics of functional analysis with Nelson Dunford, using the book *Linear Operators, Part 1*, that he wrote with Jack Schwartz. As one can see from the second volume of the book, it was in this course that I first proved something that could be called a theorem. Another course was formally given by Einar Hille and made use of his book *Functional Analysis and Semi-groups*. Hille was a consummate analyst and it was a pleasure to acquire some familiarity with various objects of classical analysis, especially the Laplace transform, from it. A third course was given by Felix Browder on partial differential equations, especially the topics of current concern to specialists. He was never well-prepared, often taking two or three runs at a proof without ever succeeding in completing it. I, however, was diligent, took my notes home and usually managed to put the collected information together to arrive at a proof, so that I gained much from his lectures.

I also read a great deal, especially from Dover paperbacks, which were available cheaply. I remember, in particular, D. V. Widder's book, *The Laplace transform*, in the Princeton University Press series, Burnside's book *The Theory of Groups of Finite Order*, and the first edition of Zygmund's book *Trigonometrical Series*, a book I read carefully. I read the second book superficially, forming the extravagant ambition of proving the Burnside conjecture on groups of odd order, proved not much later by Feit and Thompson. Zygmund's book I seemed to have read quite carefully, for it saved me from failing the oral examinations at the end of the year. I had not systematically prepared for them, thinking I could take a chance with what I knew. It turned out that I had largely forgotten what I had learned about commutative algebra in Vancouver. So things were looking bad. After algebra came analysis, and the examiner Shizuo Kakutani fortunately knew a great deal about various convexity theorems, due to one or the other of the brothers Riesz, that I too had at my fingertips—at the time, from a recent reading of Zygmund's book on trigonometrical series, but not today. So I was saved by a kind of miracle.

Sometime during the year, I solved a problem about Lie semi-groups, a topic introduced by Hille and during the summer, putting together on my own what I had learned about elliptic partial differential and about Lie semi-groups, wrote what I considered an acceptable thesis. As with my master's thesis in Vancouver, no one on the faculty could read it, so that there was some question as to whether it could be accepted. Kakutani was opposed to this, but his colleagues decided none the less to accept it. Fortunately, I did not discover any errors and much of the material was later incorporated into the book *Elliptic Operators and Lie groups* by Derek Robinson. So it had some success.

Having prepared the doctoral thesis I was completely free for the entire second year of my stay at Yale. This was one of the happiest years of my professional life. For the first time in several years, my time was my own. There was one seminar proposed on functions of several complex variables that, as it turned out, never took place because of some discord between the organizers, and lectures by S. Gaal on the paper of Selberg on the trace formula and what Godement began to refer to as Eisenstein series, a topic begun by Hecke and Maaß, a student of Hecke. I read the articles that were to be treated in the seminar on my own. By a fortunate coincidence the material on domains of holomorphy they contained allowed me to prove some relatively simple theorems on the holomorphic continuation of the Eisenstein series, a topic in Gaal's lectures which later became central for me. I forget the titles of the articles and the names of their authors.

I would like to insert here a note of appreciation to Yale for the two years I spent there as a student. After deciding to apply for admission to a doctoral program in mathematics, I applied at Harvard, Yale, and Wisconsin. Harvard accepted me, but with no money; Yale accepted me with a bursary; and Wisconsin accepted me with a position as teaching assistant. Yale was thus the best choice. What I want to admit or to explain here is how fortunate I believe I was to be spared the trial of a sudden immersion in an atmosphere of competitive and well-trained young mathematicians who had spent their undergraduate years in major centres and to have had rather two stressless years if not to make a mathematician of myself at least to have a start at it. When I arrived at Princeton after these two years my schooling was up to the more demanding style of my contemporaries, the students, and of the young faculty.

In fact my arrival at Princeton was a matter largely of chance. I would have preferred to stay at Yale and, I believe, most of the faculty would have been happy to keep me, but Kakutani was again opposed and this time successfully. The place, Princeton, and the time were chosen by hazard. Leonard Gross, then at Yale, but who later spent his career at Cornell University, suggested to me and a friend a trip to the Institute for Advanced Study, where some of his friends from his student days in Chicago, among them Edward Nelson and Paul Cohen, were spending a year. I happened to speak briefly with Nelson about my work on Lie semi-groups. It turned out that he had investigated similar topics. He was favourably impressed and, as he was to become an assistant professor at Princeton University in the following year, suggested to his future colleagues that they appoint me as an instructor. I received the appointment with no application, no letters of recommendation, nothing, only the oral recommendation of Nelson. The lives of young mathematicians, and of their older colleagues as well, in the USA, and no doubt elsewhere as well—the USA often serves as an unfortunate model far beyond its borders—are more burdened with red tape than they once were. If I had become a mathematician in the present context, domestic and international, I would, I believe, have become quite a different one and, indeed, abandoned the undertaking, or perhaps never have begun it.

Semi-groups had, however, little to do with my first years at Princeton. In part because of the work of Selberg, a good many mathematicians had returned to the study of Hecke and Robert Gunning was offering a course on his theory in Princeton which I attended. There was also a seminar on analysis, that Salomon Bochner attended and, I believe, fostered. I was asked to deliver a talk on my own work and, not having anything else to offer, discussed the somewhat accidental efforts inspired by Gaal's seminar. Bochner appreciated it, not, I would guess, so much because of the material, but because it had no relation whatsoever to my thesis, an indication of independent thought. Bochner was an analyst of broad scope, who during the early part of his career in Germany had known, I believe, Helmut Hasse, Emmy Noether and others in the Göttingen school. He encouraged me to pursue the study of these series and, thus of automorphic forms, in particular to extend the considerations from the field of rational numbers to finite extensions of this field, thus to learn about algebraic numbers. This was my first introduction to German texts, a text of Landau and that, *Vorlesungen über die Theorie der Algebraischen Zahlen*, of Hecke, and then, rather quickly as I pursued the topic of Eisenstein series, the papers of Hecke and Siegel, in which the modern theory of automorphic forms was created. Sometime during this period I also came across the monograph *The general theory of Dirichlet's series* by Hardy and M. Riesz that contains the very important theorem of Landau on Dirichlet series with positive coefficients, a theorem that was, I believe, implicit in the work of Rankin and Selberg on Ramanujan's conjecture. Rankin was a student of Hardy.

I observed in the paper *Problems in the theory of automorphic forms,* that appeared in 1970 that functoriality in my sense and the theorem of Landau taken together would yield not only Ramanujan's conjecture itself but also a very general form of it. This is a conviction I had not afterwards questioned. On writing this preface, however, I glanced again at that paper and observed that functoriality and the theorem of Landau were not in themselves sufficient; one would need in addition the principal theorem of Godement and Jacquet in their treatise *Zeta functions of simple algebras* as well as the understanding of the spectral decomposition of $L^2\big(\mathrm{GL}(n,F)\backslash\mathrm{GL}(n,\mathbf{A}_F)\big)$, available in a paper of Mœglin and Waldspurger. There is no reference to these works, both of which appeared later, in my paper. I have to return to the Godement-Jacquet theorem and understand why and how it is so strong.

After Siegel the modern theory was created, hardly entirely but I would say decisively, first by Selberg, whose trace formula, although a form of the Frobenius reciprocity theorem, was at a much more difficult analytic level, and Harish-Chandra, whose work on reduction theory, although in comparison with his work on representation theory minor, transformed the theory of automorphic forms into a part of the theory not of particular reductive groups but of all reductive groups and, ultimately, an aspect of representation theory. Certainly I, as a young mathematician, moved naturally along this route. It was inevitable; as I just observed, the trace formula, which became almost immediately after its introduction central to the theory, is intrinsically representation-theoretic.

A topic that was less obvious, but in its way also inevitable, was class field theory. Not only was it in the air in Princeton, from which Artin had not long been absent, but it was apparently also a topic that Bochner felt was important. It must have been in late summer of 1963, that he suggested that I should or declared that I must offer a course in class field theory, at the time an arcane topic, of no interest to the bulk of mathematicians. I was flabbergasted! I had hardly begun to learn algebraic number theory and the semester was about to begin. I protested that it was out of the question, but he insisted. I yielded and set about preparing the course, which placed the problem of a non-abelian class field theory, if not at the centre, certainly on the fringes of my mathematical ambition.

The academic year 1964–65 I spent at Berkeley in California with no teaching responsibilities. My initial ambition was to learn algebraic geometry. This was before Grothendieck dominated the subject and I took with me Weil's *Algebraic Geometry* and Conforto's *Abelsche Funktionen und Algebraische Geometrie*. I even organized with Phillip Griffiths a seminar on algebraic geometry from which he clearly profited much more than I. I was also infatuated during that year with Harish-Chandra's papers on spherical functions and matrix coefficients and wanted to construct that theory along the lines of the theory of hypergeometric functions with which I was taken at the time, but was unsuccessful. All in all, I was disappointed with my mathematical accomplishments during that year.

The next year, back in Princeton, was no better. I had formed two ambitions, both rather extravagant, to make some progress in non-abelian class field theory and to create a general theory of Hecke L-functions. I made no progress and by the spring of 1966 was coming to believe that mathematics was not the career for me. Indeed I had sentiments of this sort already during the year in Berkeley. The year 1963/64, a year in which I not only was learning class field theory for the course I was giving but also writing up the extremely long paper on Eisenstein series, may have caused an unrecognized exhaustion that was the source of the mathematical famine of the Berkeley year. Anyhow, as I have recounted elsewhere, a friend, Orhan Türkay whom I had met in Princeton and who was also spending a year in Berkeley, suggested I come to Turkey, not to his university where he taught economics but to the newly created Middle East Technical University. It was not a suggestion that I initially took seriously. However, after my return to Princeton and a fruitless struggle with the two topics mentioned, I recalled his invitation and began, influenced perhaps by Agatha Christie novels, to reflect on the possibility of a romantic trip to the Middle East, today a somewhat incredible notion, especially for someone accompanied by a wife and four young children.

As a diversion, a pleasant one, during 1965–66 I offered to teach mathematics for engineers, a task that was beneath the dignity of my colleagues. I enjoyed it and the engineering students and I had, as well, the pleasure of consulting various books that appealed to me. From Maxwell's *Electricity and Magnetism* I learned methods for plotting the level lines of harmonic functions, an amusing occupation for me and for the students; Relton's book on Bessel functions was an opportunity to learn some concrete spectral theory. I do not suppose that specialized monographs were better or more readily available fifty or sixty years ago than today, but I was more likely to find the time, the energy, and the patience to read them.

Once having begun to make the arrangements for a year, or more, abroad, life became simpler and more relaxed. For lack of anything better to do, I began, perhaps in the summer or the autumn, to make some idle calculations of the constant terms of the Eisenstein series. Once calculated, it was, as I recall, almost immediately evident that they offered examples of a general notion of a Hecke L-function, what I had been searching for in vain. I did not change my plans, but did abandon my efforts to learn Turkish and to improve my knowledge of Russian, both appealing pastimes that had to be abandoned in order to investigate the implications of these calculations. By the end of the year I had a fairly clear idea of what might be done and, during a brief, unexpected conversation with André Weil began to explain what I had in mind. A more detailed handwritten explanation sent after the conversation he did not find persuasive, but he did invite me to attend a seminar in which he was explaining how he hoped to extend the Hecke theory for GL(2) to general number fields. He was having trouble with the complex case. I explained the theory in this case to him in two letters, one sent before the departure for Turkey, one after my arrival, in which, thanks to the knowledge of ordinary differential equations acquired over the years and to my own experiences in attempting to create a generalized Hecke theory, the complex case was treated. Weil was unable to read them, but turned to Jacquet for help, and this led to the joint effort with Jacquet to develop a theory of automorphic forms for GL(2) compatible with my expectations, and perhaps with Jacquet's as well.

Apart from childhood excursions across the nearby border and my years at Yale, Princeton, and Berkeley, I had never been abroad, never visited French-speaking Canada, so that Turkey was my first experience with a genuinely different country. Although it was a pleasant and instructive visit that did not interfere with my mathematical projects, I did not accomplish what I had hoped, scarcely, in spite of my intentions, learned the language, scarcely visited the country and acquired only a superficial understanding of its history. My first mistake was not to take an immediate linguistic plunge, not to understand that for an anglophone English is, in many respects, a handicap in one's encounter with the world.

The language itself attracts too many people and the wrong sort; it alienates others and is, by and large, an obstacle to a genuine intimacy with a land and its people. I exaggerate but for, say, an academic to acquire English today is scarcely more difficult than learning to drive an automobile, yet there are many academics, in particular, a good number of my fellow mathematicians, who regard it as a genuine intellectual accomplishment, which they are eager to display whenever the occasion arises. They and, in particular, their excessive use of English as a vehicle of publication deprives the field of a great deal of whatever secondary intellectual pleasures it offers. Before my first visit to Turkey, I did not understand this. The situation was not so extreme in 1967 as it is today. At all events during this visit I acquired some knowledge of the language, some acquaintances among the students that, when many years later, more experienced, better prepared, I returned, served me well. A number of the former students became friends.

In the meantime, I had been more determined in my efforts, linguistic and historical, but since my major preoccupation was always mathematics, I was not able to satisfy these predilections to any satisfactory extent. I have presently one final mathematical project, but once it is accomplished, or even if I make some encouraging progress, I hope to pass the time left to me indulging them. My memory is failing; so are my energies, but I am nevertheless hopeful.

The major undertaking of my mathematical career has been the theory of automorphic forms and its many manifestations, and a secondary one has been renormalization. I think that most of my colleagues regard me as competent to discuss automorphic forms; very few will regard me as competent to discuss renormalization. I shall nevertheless attempt to do so very briefly, because I fear it is a subject with enormous potential and enormous depth although also enormously difficult that has not been met with any genuinely adequate effort on the part of mathematicians. Here, however, as with automorphic forms, the principal failing seems to be a reluctance to come to terms with the central issues, to forget that the central problem in both cases is to construct a theory, not to prove something that with a certain amount of good will passes muster as a theorem.

Automorphic forms are another matter, for the major problems have not been entirely neglected, but there is still much more that we do not understand than that we understand. At present, the subject has strong ties to arithmetic, to analytic number theory, to algebraic geometry, and to differential geometry. There are three different theories, relevant in three different contexts: the theory over an algebraic number field; the theory over a function field in one variable over a finite field; the theory over a compact Riemann surface, thus over a function field in one variable over $\mathbf{C}$. They have many similarities, but a number of important differences. There is a famous analogy of earlier forms of them, in which reductive Lie groups, apart from GL(1), played no role, with the Rosette Stone. Weil has written a brief and charming description of it in the essay *De la métaphysique aux mathématiques*, but the analogy is less persuasive for the modern theories. The essay is none the less well worth reading. There is an extension of the analogy in a Bourbaki lecture of Edward Frenkel, *Gauge theory and Langlands duality,* that appeared in a Bourbaki lecture in 2009. He has our three topics, which he lists as *Number theory, Curves over* $\mathbf{F}_q$, *Riemann surfaces* but he adds a fourth, *Quantum Physics.* I prefer to remain in this essay within the domain of pure mathematics, indeed lack of competence forces me to do so.

I shall be discussing the last of the three topics at more length in an article Об аналитическом виде геометрической теории автоморфных форм but for now I prefer to confine myself to a very superficial review of the three theories mentioned. It is well to recall that the first theory, which is central to the theory of algebraic numbers and diophantine equations, has a history beginning late in the eighteenth century or early in the nineteenth, with Gauss and Kummer and continuing without interruption until the middle of the last century; the second, which may ultimately form a major part of the theory of finite fields, is thus a theory that begins perhaps with Galois, but began to play a much larger role with the introduction by Hasse and Weil of an algebraic geometry over finite fields; the third is at best a part of the theory of Riemann surfaces, which itself does not, as one remarks when recalling Euler's concern with elliptic integrals, begin entirely with early nineteenth century mathematicians like Abel, Jacobi, Weierstrass, and Riemann and their study of algebraic curves or Riemann surfaces, but acquires at that time a new richness. It also entails—even without turning to the fourth, neglected topic—a relation to, perhaps speculative, physical theories, the introduction—in the definition of the geometric form of the Hecke operators—of differential geometry, curvature, and the Chern-Gauss-Bonnet theorem, which appears—in spite of its name—to be, in its present form, largely a twentieth century creation. I, myself, greatly regret as my career draws to a close not having spent enough time with the writings of the early founders of these theories.

It is certainly not necessary to understand the theories as a whole to contribute to the theory of automorphic forms. On the other hand, deliberately to restrict one's attention to a few tried methods and to a few familiar concepts, and deliberately to turn one's back on a marvellous fusion and coherence of many of the principal mathematical concepts of the past 250 years, seems to me an unpardonable form of self-mutilation, rendered more tempting by the linguistic, thus intellectual, inadequacies noted above. For many, even most, contemporary mathematicians, the mathematical past is largely inaccessible. This ignorance is unfortunate. Humans have not been on the earth for such a long time that it is appropriate to forget our history, or that of our accomplishments, mathematics among them. The indifference to the mathematical past and the distance from it will, I suppose, nevertheless become more pronounced if or when the Asian nations assume a major role in mathematics and even English ceases to be the principal, or even an adequate, medium of communication. The possibility of reducing mathematics to a trivial pursuit, a struggle for a large number of citations, for a prize, or just for a tenured position, is also there. It is difficult not to be pessimistic! In my view, or at least in a coarse simplification of my view, a great deal has been lost or destroyed in the largely successful European—at least initially, for the centre of power shifted in the last century—attempt to conquer or dominate the world and little is done to preserve what remains.

To return to the three theories. I have spent most time with the first and believe that it is the most difficult, although it may now have the fewest practitioners. The two principal questions are very general: functoriality and reciprocity. Functoriality entails an answer to such questions as Ramanujan's conjecture in its general form, thus it entails establishing all the expected properties of automorphic L-functions short of the Riemann hypothesis. A first tool is the trace formula in the form established and developed extensively by James Arthur. Then it requires a development of the trace formula, in the sense of analytic number theory, say in the form with which Ali Altuğ has been struggling, whose goal would be, in my view, the formulation and the proof of the diophantine equalities, thus a comparison of the number of solutions of two different diophantine equations, necessary for the comparisons envisaged in my essay *A prologue to functoriality and reciprocity: Part*

1. This is not work that I myself have undertaken, perhaps because I have no ideas, but largely because I think of it as an undertaking that requires decades and I myself do not have decades. Once functoriality has been established or some progress with it has been accomplished, there will remain reciprocity: is the motivic galoisian group a quotient of the automorphic galoisian group? This is an even more daunting problem. There may be domains and mathematicians whose efforts suggest solutions but my ignorance is so great that I am reluctant to offer any advice. Otherwise, we have only scraps of results, scraps of a method, but they are serious scraps, prominent in, for example, the proof of the Fermat conjecture. I do not suppose we shall ever have any general information about these two groups more concrete than a statement that one is a quotient of the other. Specific information is another matter.

In the article in preparation whose title appears above, I broach the problem of an explicit description of the automorphic galoisian group for the third form of the theory of automorphic forms, thus the theory over a Riemann surface. There is no analogue of the motivic galoisian group. I confine myself in the article to unramified representations, thus to a nevertheless large and interesting quotient of the galoisian group, and define a group that I call the AB-group because it was introduced by Atiyah and Bott and make some effort to persuade the reader that it is indeed the unramified automorphic galoisian group for the third theory. Further reflection is, however, necessary.

I have not yet had a chance to examine the geometric theory over a finite field, but, after hearing of the work of Vincent Lafforgue, I am tempted to ask myself whether the automorphic galoisian group over function fields over a finite field κ is not pretty much the Galois group of the function field F_κ, the field of rational functions on the curve defined over κ, although I have not yet had the leisure to study his papers.

For a period of several years, I was absorbed by an altogether different mathematical theory. Sometime in 1984/85 I had the good fortune of an enlightening conversation with the physicist Giovanni Gallavotti. As we strolled on the grounds of the Institute for Advanced Study, he described to me the mathematical problems arising from renormalization. They are very beautiful, of very general concern, and, in some sense, not sufficiently studied by mathematicians. There is, as I discovered, a great deal to learn if one wishes to understand their significance, whether in quantum field theory or fluid dynamics, or even in a more mathematical context. I was never able to come to grips with all these matters, although a colleague Yvan Saint-Aubin and I spent a great deal of time with numerical experiments. Once again I hope, after I have succeeded in explaining what I see as the expression of functoriality in the automorphic theory over (the function fields of) Riemann surfaces, to return to renormalization, not in the expectation of accomplishing anything, but rather in the desire to understand the problems in more depth than before.

Let me try to describe the relevant issues in a very simple, somewhat factitious example. I am hardly in a position to discuss quantum field theory or fluid mechanics. Renormalization is related to a change of scale. Suppose we have a cube (my personal experience is limited to dimension two) of porous material whose precise constitution is unknown, although we know the probability—referred to as a crossing probability—that water forced into the material on one small area α on the surface can make its way to another small area β, thus that there is an open channel between them. The collection of crossing probabilities $\{\pi(\alpha, \beta)\}$ is a property of the material used. An extreme case would be that the initial choice was between a solid, thus impassable, cube with probability x and an empty cube, in which all crossings are possible, with probability $1 - x$. We can then think of taking eight of such cubes and placing them together to form a cube with double the original linear dimension. It will have different crossing probabilities, because in the larger cube, they are affected by the possibility of a very large number of paths, moving in and out of the eight constituent cubes. Then we change scale so that the new cube has edges of length one, thus so that the original cube has sides of length $1/2$. We continue in this way. Thus we start with something very small and perhaps very simple, as nature seems to do, and arrive at something very large. What happens? We can expect that most of the time, thus for most initial probability distributions, as the size grows, complete permeability or complete impermeability become more and more likely. On the other hand, it may be that some other configurations maintain themselves or are generated in the process. There are of course an infinite number of initial distributions, even an infinite-dimensional space of initial distributions. Nevertheless there may be a tendency for all of it, or for large chunks of it, to be shrunk by an infinite number of repetitions to a point. These points and their nature are of considerable interest. What happens, for example, if we start from a point in the vicinity of one these limit points. Typically, there is a subspace of finite codimension that contracts under the operation described to the point while there is an expansion in the remaining directions, so that each orbit under repeated applications of the process described forms a kind of discrete hyperbola, except for those that start on the subspace. These dynamics are very simple.

It is, however, not even well understood how to create such systems. More to the point it is not well understood, certainly not in a mathematical sense, how to create such systems that are physically relevant nor how to demonstrate that some system, presumed to be physically relevant, has the expected mathematical properties. It would, I think, be a pleasure to reflect on these matters, to try to understand some part of what is known and something of what is not known, but with absolutely no ambitions.

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