First some remarks on groups $\mathcal{P}$ acting on a product space $S$, call reducible if $\mathcal{P}$ commensurable with $L$ has common subgroup of finite index with a direct product $\mathcal{P}_1 \times \mathcal{P}_2$ where $\mathcal{P}_1$ acts on $S_1$, $\mathcal{P}_2$ on $S_2$, and $S = S_1 \times S_2$ otherwise "irreducible". The irreducible groups on $S$ are in a sense the only groups the properly belong to that space. Consider only those irreducible: (1) projection of $\mathcal{P}$ on any factor $G_i$ of $G$ is everywhere dense in $G$, (2) if $g$ in $\mathcal{P}$ projects in identity on any factor $G_i$ of $G$ then $g = e$.

I shall for simplicity first consider product of two copies of SL(2, $R$) (by the standard product of two hyperbolic planes) not much diff' if one factor is SL(2, $C$) or hyp. 3-space. Assume group $\mathcal{P}$ acting on $H^2$ has no element outside of finite order, (otherwise could go to subgroup of finite index). Volume $V$ finite and for time being assume $D$ compact. If it is easy to show that in this case $V(B) = \text{integer}$.

Consider for integer $\ell \geq 1$

functions in $z = (z_1, z_2)$ which under transform in way

$$y \ z = \left( \frac{a_1 z_1 + b_1}{c_1 z_1 + d_1}, \frac{a_2 z_2 z_1 + b_2}{c_2 z_2 z_1 + d_2} \right)$$

$$F(y \ z) = \frac{(c_2 z_1 + d_1)^\ell}{c_1 z_1 + d_1} F(z), \text{and}$$

which furthermore are such that

$$\partial_{\frac{a_i}{c_i^{\ell}}} F(z) = 0 \text{ for } a_i F(z) \text{ analytic in } z_1,$$

$$z_2$$
Form the kernel
\[
\left( \frac{1}{2_1 - \xi_1} \right)^2 \frac{1}{2} \left( \frac{12 - 2 \xi_1^2}{2^2 y_2} \right)
\]
where \( k \) is a function tending reasonably
to zero, say as \( t \to \infty \), \( k(t) = O \left( \frac{1}{t^{1+\epsilon}} \right) \).

This for \( \lambda > 1 \) is an admissible
kernel and we can at once
write down the resulting basic
formula for \( \Gamma \), since the various
integals that occur split into
direct products of two integrals
of the kind that occur in a single
copy of \( \Gamma \).

Denoting by \( \lambda_i \) the values for
which
\[
y_2 + \lambda_i F(z) + \left( \frac{1}{4} + \lambda_i^2 \right) F(z) = 0
\]
has a square integrable solution \( \lambda_i \) defines
let \( \mu(z) \) and \( \eta(z) \) be the functions
which derive from \( k \) in the way
stated in my lectures here in 1956,
we can now write down the formula
(1) only elements \( \mu \) which contribute
are those where first component of
\( \mu \) is elliptic, second hyperbolic,
primitive
Denote the equivalence classes
by \( \Sigma F \), call them primitive
of not positive power with \( \exp \gamma \) of
other element in group.
prim class described by two parameters $\alpha, \phi$.

The minimal distance $p$ that the element moves a point in second copy of $\mathfrak{H}$.

Get

$$\sum_{j=0}^{\infty} b(n_j) = \frac{V(D)}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-y}}{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}} dy$$

$$\sum_{j=0}^{\infty} \sum_{m=1}^{\infty} \frac{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}{e^{-\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}} q(m, p)$$

$$\sum_{j=0}^{\infty} b(n_j) = \frac{V(D)}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-y}}{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}} dy$$

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Structure quite similar to formula for a single hyperplane only factor $\frac{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}{1 - e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}$ seems in convenient. Combine with $\gamma^{-1}$ change $p$ but conjugate $\gamma$'s.

and

$$\frac{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}{1 - e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}} = - \frac{e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}{1 - e^{\frac{\alpha}{2} - \frac{\alpha}{2} + i\phi}}$$

$$= - \sum_{|z| < \varepsilon} e^{z i}$$

Form

$$Z(e^0, P) = \prod_{\gamma > 0} (1 - e^i - (\alpha + \gamma) P)^{-1}$$

has analytic cont. in integral function with functional eqn.
\[ Z(1-S) = Z(1) \exp \left(-\frac{(k-1) \text{V}(D)}{4 \pi \text{y}^2} \int_0^\infty \text{d}y \right) \]

zeros in critical strip at \( \frac{1}{2} + in \) and

trivial zeros at negative integers \(-n\)

no poles.

For \( l = 1 \) kernel is not

admissible not in \( L_2 \) Formally

if we put \( l = 1 \) above we get

\[ Z(1) = \frac{1}{\sqrt{\pi}} \left(1 - e^{-2\pi y} \right)^{-1} \]

\( y > 0 \)

which clearly cannot have a singularity

somewhere on the real axis, so something

will have to change.

For \( l = 1 \) we modify our kernel by

introducing a convergence factor

\[ \left( \frac{\text{y}^{1-n}}{1^{2-l} - \text{y}^{1-l}} \right)^x \]

with \( x > 0 \)

our eigenfunctions are now eigenfunctions

of two operators

\[ q_1 \Delta_1 - 2 \pi y_1 \frac{\partial}{\partial y_1} \]

and \( q_2 \Delta_2 \)

in the above formula we get for this

we could try to let \( x \to 0 \), this would

be extremely messy. Instead

we also write down the T formula

from kernel

\[ \frac{\alpha}{1+\alpha} \left( \frac{q_1 y_1}{1^{2-l} - \text{y}^{1-l}} \right)^{1+\alpha} \left( \frac{y_2 - y_1}{y_2 - y_2} \right)^{1+\alpha} \Delta_2 \]
The spectra in the two cases are
greatly identical, except that the
greenwich with \( v_1^+ F(z) \) holomorphic in \( z \),
not occur in second formula, and
the point corresponding to the constant
eigenfunction does not occur in the first
on other side also must terms drop
out when taking the difference. After
dividing with \( \gamma \) the resulting formula
for \( \lambda = 1 \) is

\[
\sum_{\xi} \frac{\partial}{\partial \xi} \left( \frac{\xi}{\xi + z} \right) \psi_{i,\xi}(x) \psi_{i,\xi}(y)
\]

We see that statements about \( \psi(x) \)
remain but about \( \bar{Z}(x) \) except that
\( \bar{Z}(x) \) has a pole of order 1 as \( s = 1 \).

Product of \( n \) hyperplanes use special
terms for first \((n-1)\) factors

\[
\prod_{l=1}^{n} \left( -\frac{1}{\xi_l} \right)^{\mu_l} \psi_{\mu_l}(x) \psi_{\mu_l}(y)
\]

for \( \mu_l = \mu_{l-1} = 1 \); have pole or zero at \( s = 1 \)
depending on whether \( n \) is even or odd.
Angles equidistributed with respect to measure $\frac{2\pi}{2\pi} = 1 - \cos \varphi$

and statistically independent of hybrid cases.

$|Z_1|^2 + \ldots + |Z_m|^2 < 1 \quad m > 1$

and also