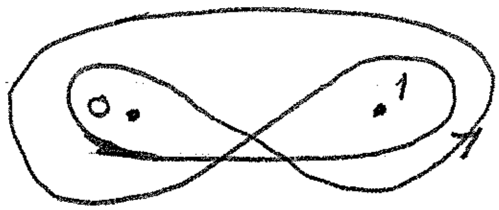


$$\int_{\mathcal{C}} \int_{\mathcal{C}} \cdots \int_{\mathcal{C}} (t_1 \cdots t_n)^{x-1} \left((1-t_1)\cdots(1-t_n)\right)^{y-1} \left[\cancel{dt}, (t_1 \cdots t_n)\right]^{2m} dt_1 \cdots dt_n$$

$$= \left(4 \sin \pi x \sin \pi y\right)^{n} \cdots$$

remarks on coeff. & boundary cases.

Applications

own in 1941 used to determine a linear combination of

$$\sum_{h=1}^{n} \sum_{m=0}^{M} \frac{a_m^h}{(z-m)^h}$$ max small on some circle

with radius $R$ = ~~$cM/2$~~ (with $c$ somewhat larger than $M$.

In diff. Bombieri in connection with Tchebicheff's.

one is led to correct formulation
let $P_2$ run over all quadratic polynomials $x^2+ax+b \pmod p$, $D(P) =$ discriminant.

$$\sum_{P_2(p)} \chi_1(P(0))\, \chi_2(P(1))\, \chi_3\psi(D(P_2))$$

conjecture for general $n$
$P_n(x) = x^n + a_1 x^{n-1} + \cdots + a_n$ (,

$$\sum_{P_n \pmod p} \chi_1((-1)^n P_n(0))\, \chi_2(P_n(1))\, \chi_3\psi(D(P_n))$$

$$= \prod_1^n \frac{\tau(\chi_3^\nu)}{\tau(\chi_3)} \frac{\tau(\chi_1\chi_3^{(\nu-1)})\,\tau(\chi_2\chi_3^{\nu-1})}{\tau(\chi_1\chi_2\chi_3^{n+\nu-2})}$$

certain exceptions. $\bar{\chi}_1\chi_2\chi_3^{n+\nu-2} \neq 0$

Cannot prove at present for $n > 2$.

Similarly analog of (4) and 5 can be written out. (no limit procedure is known)

$$\sum \chi_1(h_1, h_2]\, \varepsilon^{h_1+h_2}\, \chi_3^2(h_1-h_2)$$

the better analog.

or

$$\sum \chi_1(P_2(0))\, \varepsilon^{P_2'(0)}\, \chi_3\psi(D(P_2))$$

generally

$$\sum \chi_1((-1)^n P_n(0))\, \varepsilon^{\frac{1}{(n-1)!}P_n^{(n-1)}(0)}\, \chi_3\psi(D(P_n))$$

and

$$\sum \varepsilon^{h_1^2+h_2^2}\, \chi_3^2(h_1-h_2)$$

~~general~~

$$\chi_1((-1)^n P(0))\ \varepsilon^{a_1^2 - a_2}\ \chi_3 \psi(D(P_n))$$

can prove formula for this also for $n = 3$, but no further at present.

History.

M. L. Mehta: Random Matrices & Statistical theory of energy levels. Acad Press N.Y. 1967

also papers by Dyson & Dyson–Mehta.

S. Karlin & L. S. Shapley. Geometry of Moment Spaces. Mem. Am. Math. Soc 12 Prov. RI 1953

S. S. Wilks Math. Statistics Wiley N.Y. 1962.

grense fall.

(4)
$$\int_0^\infty \cdots \int_0^\infty (t_1 \cdots t_n)^{x-1} e^{-t_1 - \cdots - t_n} |\Delta(t)|^{2z} dt_1 \cdots dt_n$$

$$= \prod_{\nu=1}^{\infty} \frac{\Gamma(1+\nu z)}{\Gamma(1+z)} \Gamma(x+(\nu-1)z)$$

(5)
$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t_1^2 + \cdots + t_n^2)} |\Delta(t)|^{2z} dt_1 \cdots dt_n$$

$$= (2\pi)^{\frac{n}{2}} \prod_{\nu=1}^{n} \frac{\Gamma(1+\nu z)}{\Gamma(1+z)}$$

also f-els $\mu$ real.

(6)
$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{|\Delta(t)|^{2z} e^{i\mu(t_1+t_2+\cdots+t_n)}}{((\omega+it_1)\cdots(\omega+it_n))^{x}} dt_1 \cdots dt_n$$

Arbeide fra 1940 grense-

approksimasjons problemet.

Skissere bevis av (1).

anta $z$ = helt tall $m \geq 0$

$$|\Delta(t)|^{2m} = \sum c_{\alpha_1 \cdots \alpha_n} t_1^{\alpha_1} \cdots t_n^{\alpha_n}$$

anta $\alpha_1 \leq \alpha_2 \cdots \leq \alpha_n$

klart at $\alpha_n \geq (n-1)m$; og like

$$\alpha_\nu \geq (\nu-1)m$$

og at $(|\Delta(t)|)^{2m} = (t_1 \cdots t_n)^{2(n-1)m} |\Delta(\frac{1}{t})|^{2m}$

hvorav. $\alpha_\nu \leq 2(n-1)m - (n-\nu)m =$

$$(n+\nu-2)m$$

$$(\nu-1)m \leq \alpha_\nu \leq (n+\nu-2)m$$

Eulers formel.

$$\frac{P(x+\alpha_\nu)\,P(y)}{P(x+y+\alpha_\nu)} = \prod \frac{P(x+(\nu-1)m)\,P(\cdot}{P(x+y+(n+\nu-2)m)} \cdot P(y) \cdot P_1(x,y)$$

$$= \prod \frac{P(x+(\nu-1)m)\,P(y+(\nu-1)m)}{P(\cdots)} \frac{P_1(x,y)}{((y+1)\cdots y+(\nu-1)m)}$$

$P_1$ grad i $y$

höist same som nevner.

$$\frac{P_1(x,y)}{\cdots} = \frac{P_1(y,x)}{(x+\cdots x\cdots}$$

kan slutte at integralet er

$$c_n(m) \prod \frac{P(x+(\nu-1)m)\,P(y+(\nu-1)m)}{P(x+y+(n+\nu-2)m)}$$

Sett $x = y = 1$

$$\int_0^1 \int_0^1 (|\Delta(t)|)^{2m} dt_1 \cdots dt_n = n \int_0^1 dt_n \int_0^{t_n} \cdots \int_0^{t_n} |\Delta_{n-1}(t)|^{2m} \cdot (t_n-t_1)^{2m} \cdots$$

$dt$

Sett. $t_j = t_n t_j'$ for $1 \leq j \leq n-1$

får rekursivt bestemt $c_n(m)$ og da

$$\int_0^1 \cdots \int_0^1 |\Delta_n(t)|^{2m}\, dt_1 \cdots dt_n = \frac{1}{1+}$$

$c_1(m) = 1$ noe regning gir

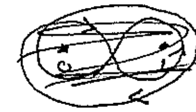
$$c_n(m) = \prod_{\nu=1}^{n} \frac{\Gamma(1+\nu m)}{\Gamma(1+m)}$$

vist for heltallig $z$. Sammenlignes funksjoner for $z \geq 0$ begge begrenset i halvplan. og diff $= 0$ for $z = 1, 2, 3 \ldots$ ; derfor kan lett vise identisk lik $0$.

for $R(z) > a$. kan

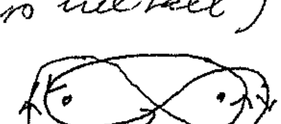
$$f(z) = \frac{1}{2\pi i} \int \frac{f(\zeta)\, \cancel{z}(z-1)\cdots(z-a)}{(\zeta - z)\cancel{\zeta}(\zeta-1)\cdots(\zeta-a)}\, d\zeta$$

endelig kan slutte hvordan til nærmeste singularitet i $z$.



Bemerkninger om komplekse integraler

kombinatoriske problemer (for ~~...~~ pro heltall).



Analoge Karaktersummer.

analogi av $\Gamma$ funksjon

$$\tau_\chi = \sum_{h \,(\mathrm{mod}\, p)} \chi(-h)\, \varepsilon^h$$

for $\sum_{h_1, h_2} \chi_1(h_1 h_2)\, \chi_2((1-h_1)(1-h_2))\, \chi_3^2(h_1 - h_2)$

$\varphi$ er kvadratisk karakter.

$$= \frac{\tau_{\chi_3^2}}{\tau_{\chi_3}} \cdot \frac{\tau_{\chi_1}\, \tau_{\chi_1\chi_3}\, \tau_{\chi_2}\, \tau_{\chi_2\chi_3}}{\tau_{\chi_1\chi_2\chi_3}\, \tau_{\chi_1\chi_2\chi_3^2}} \qquad (*)$$

$$+ \frac{\tau_{\chi_3^2}}{\tau_{\chi_3\varphi}} \quad \frac{\tau_{\chi_1}\, \tau_{\chi_1\chi_3\varphi}\, \tau_{\chi_2}\, \tau_{\chi_2\chi_3\varphi}}{\tau_{\chi_1\chi_2\chi_3\varphi}\, \tau_{\chi_1\chi_2\chi_3^2}} \qquad (**)$$

enklere
lignende formel for

$$\sum \chi_1(h_1 h_2)\, \varepsilon^{h_1 + h_2}\, \chi_3^2(h_1 - h_2)$$

og $\sum_{h_1, h_2} \varepsilon^{a(h_1^2 + h_2^2)}\, \chi_3^2(h_1 - h_2)$

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Mere perfekt analogi om man summerer for alle 2 grads polynomer mod $p$ $x^2 + ax + b$; rötter $\alpha, \beta$

$$\sum \chi_1(\alpha\beta)\, \chi_2((1-\alpha)(1-\beta))\, \chi_3\varphi((\alpha-\beta)^2)$$

får $= (*)$.

ledet til å tentativt anta at rette generalisasjon av $P_n(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n$

$$D_n = \prod_{i<j} (\alpha_i - \alpha_j)^2$$

$\sum$ ~~$\chi_1(a_n)\chi$~~

$(P_n(x) \to (-1)^n P_n(1-x))$

~~$\sum \chi_1((-1)^n P_n(0))\chi_2(P_n(1$~~

$$\sum \chi_1((-1)^n P_n(0))\,\chi_2(P_n(1))\,\chi_3\psi(D_n))$$

$$= \prod_{\nu=1}^{n-1} \frac{\tau_{\chi_3^\nu}}{\tau_{\chi_3}} \frac{\tau_{\chi_1 \chi_3^{\nu-1}}\ \tau_{\chi_2 \chi_3^{\nu-1}}}{\tau_{\chi_1 \chi_2 \chi_3^{n+\nu-2}}}$$

$(\chi_1 \chi_2 \chi_3^{n+\nu-2} \not\equiv \chi_0)$.

hilsen fra andre enklere summer

Ronald Evans. 1980 San Diego.

hørte foredrag nevnte, kom frem til samme formodning og regnet ut for $n = 3$ og $4$ og mange små primtall. Slemte

Bevis må kreve helt nye ideer. Også de andre fall krever nevnte, kan ikke gjøre grense-overgang.