Lecture 9

THE MODIFIED BASIC IDENTITY AND WEIGHTED ORBITAL INTEGRALS

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9.1. The modified basic identity.

As was pointed out in Lecture 5, it seems to be more convenient when dealing with σ-expansion to use a completely ε-invariant truncation. Let P be an ε-invariant parabolic subgroup of G; we define ${}_\varepsilon\hat{\tau}_P$ to be the characteristic function of the set of $X \in \mathfrak{a}_0$ such that

$$\varpi_{\bar{\alpha}}(X) > 0 \qquad\qquad \forall \alpha \in \Delta_0 - \Delta_0^P$$

where by definition

$$\varpi_{\bar{\alpha}} = \frac{1}{\ell} \sum_{r=0}^{\ell-1} \varepsilon^r \varpi_\alpha$$

and $\bar{\alpha}$ is the orbit of α under E.

We need the analogue of Lemma 2.1, namely the

LEMMA 9.1.1. <u>There are constants</u> c <u>and</u> N <u>such that the number of</u> $\delta \in P\backslash G$ <u>for which</u>

$$\varpi_{\bar{\alpha}}(H(\delta x) - T) > 0$$

<u>for all</u> $\alpha \in \Delta_0 - \Delta_0^P$ <u>is at most</u> $c|x|^N e^{N\|T\|}$.

We need only to show that we can find a set of representatives for these δ each of which satisfies

$$|\delta| < c'|x|^{N_1} e^{\|T\|N_1} .$$

According to reduction theory we may choose δ such that $\delta x \in \mathfrak{S}_P(T_0)$ and more precisely we may assume that $\delta x = nam$ with $n \in \omega_1$ a compact set in $\mathbf{N}$, $m \in \omega_2$ a compact set in $\mathbf{G}$ and $a \in A_0(\mathbb{R})^o$; moreover $H(\delta x) = H(a)$ and

$$(1) \qquad \alpha(H(\delta x)) > \alpha(T_0) \qquad \forall \alpha \in \Delta_0^P .$$

Taking over the numbering of Lecture 2, we have

$$(3) \qquad \varpi_\alpha(H(\delta x)) \leq c_1(1 + \log|x|) \qquad \forall \alpha \in \Delta_0 .$$

Our assumption is not (4) as in Lecture 2 but only

$$(4') \qquad \varpi_{\bar{\alpha}}(H(\delta x)) > \varpi_{\bar{\alpha}}(T) \qquad \forall \alpha \in \Delta_0 - \Delta_0^P .$$

But (3) and (4') imply

$$(4'') \qquad \varpi_\alpha(H(\delta x)) > \varpi_{\bar{\alpha}}(T) - \frac{\ell - 1}{\ell} c_1(1 + \log|x|)$$

for any $\alpha \in \Delta_0 - \Delta_0^P$.

The inequalities (1), (3), and (4) yield the inequality

$$(2) \qquad \|H(\delta x)\| < c_2(1 + \log|x| + \|T\|) .$$

Since $\delta x = nam$ with n and m in compact sets and $H(\delta x) = H(a)$ we conclude that

$$\log |\delta x| \leq c_3(1 + \log |x| + \|T\|) \quad . \qquad \square$$

Given $P_1 \subset P \subset P_2$ three parabolic subgroups with P ε-invariant we define ${}_P\sigma_1^2$ to be the characteristic function of the set of $H \in \mathfrak{a}_0$ such that

(i) $\alpha(H) > 0 \qquad \forall \alpha \in \Delta_1^2$

(ii) $\alpha(H) \leq 0 \qquad \forall \alpha \in \Delta_1 - \Delta_1^2$

(iii) $\varpi_{\bar{\alpha}}(H) > 0 \qquad \forall \alpha \in \Delta_0 - \Delta_0^P$.

We obviously have

$$\sum_{P_2 \supset P} {}_P\sigma_1^2 = \tau_{P_1}^P \, {}_\varepsilon\hat{\tau}_P \quad .$$

Given P_1 and P_2 we define ${}_\varepsilon\sigma_1^2$ to be zero if there is no ε-invariant parabolic subgroup between P_1 and P_2 and to be ${}_Q\sigma_1^2$ if Q, the maximal ε-invariant parabolic subgroup in P_2, contains P_1.

LEMMA 9.1.2. _If_ $P_1 \subset P \subset P_2$ _with_ P _ε-invariant then_

$${}_P\sigma_1^2 = {}_\varepsilon\sigma_1^2 \quad .$$

Clearly ${}_P\sigma_1^2 \leq {}_\varepsilon\sigma_1^2$; now consider H in the support of ${}_\varepsilon\sigma_1^2$, we need to prove that for any $\alpha \in (\Delta_0 - \Delta_0^P) \cap \Delta_0^Q$ we have $\varpi_{\bar{\alpha}}(H) > 0$.

We know (cf. Lecture 2, page 7) that

$$\varpi_\alpha = \varpi'_\alpha + \sum_{\beta \in \Delta_0 - \Delta_0^Q} \lambda_{\alpha\beta}\varpi_\beta$$

with $\lambda_{\alpha,\beta} \geq 0$ and $\varpi'_\alpha \in \hat{\Delta}_P^Q \subset \hat{\Delta}_1^Q$. The same equation holds for averaged weights, and by assumption $\varpi_{\bar{\beta}}(H) > 0$. All we need to prove is that $\varpi'_\alpha(H) > 0$, but $\Delta_1^Q \subset \Delta_1^2$ and hence $\gamma(H) > 0$ for any $\gamma \in \Delta_1^Q$; using assertion (c) of Lecture 2, page 6, which tells us that

$$(\mathfrak{a}_1^Q)^+ \subset {}^+\mathfrak{a}_1^Q$$

we see that $\varpi'_\alpha(H) > 0$ for any $\varpi'_\alpha \in \hat{\Delta}_P^Q$. □

We may now state the modified basic identity

PROPOSITION 9.1.3. Given P, an ε-invariant parabolic subgroup we have

$$\sum_{\delta \in P\backslash G} K_P(\delta x, \delta x)\, {}_\varepsilon\hat{\tau}_P\,(H(\delta x) - T)$$

$$= \sum_{P_1 \subset P \subset P_2} \; \sum_{\xi \in P_1\backslash G} {}_\varepsilon\sigma_1^2(H(\delta x) - T)\,\Lambda^{T,P_1} K_P(\xi x, \xi x) \quad .$$

Using Lemma 2.4 (in Lecture 2) one needs only to remark that, thanks to Lemma 9.1.2 we have

$$\sum_{P \subset P_2} {}_\varepsilon\sigma_1^2 = \tau_1^P \, {}_\varepsilon\hat{\tau}_P \quad .$$

□

In Lectures 3 and 4 we may now substitute ${}_\varepsilon\hat{\tau}_P$ for $\hat{\tau}_P$ and ${}_\varepsilon\sigma_1^2$ for σ_1^2, and no change in the proofs is needed except Lemma 4.2.2 which has to be replaced by

LEMMA 9.1.4. Assume that $H \in \mathfrak{z}^\varepsilon \backslash \mathfrak{a}_1^\varepsilon$ and $X \in \omega$ (some compact set in $\mathfrak{a}_0$) are such that ${}_\varepsilon\sigma_1^2(H-X) = 1$; then there exist a constant c independent of X such that

$$\|H_2\| < c(1 + \|H_1\|) \quad .$$

Recall that $H = H_1 + H_2$ is the decomposition associated with the direct sum

$$\mathfrak{z}^\varepsilon \backslash \mathfrak{a}_1^\varepsilon = (\mathfrak{a}_R^Q)^\varepsilon \oplus \mathfrak{z}^\varepsilon \backslash \mathfrak{a}_2^\varepsilon \quad .$$

By assumption $\alpha(H-X) \leq 0$ for any $\alpha \in \Delta_1 - \Delta_1^2$ and then

$$\alpha(H_2) \leq -\alpha(H_1) + c_1$$

for any $\alpha \in \Delta_R - \Delta_R^Q$, and some constant c_1.

For any $\alpha \in \Delta_0 - \Delta_0^Q$ we have assumed that $\varpi_{\bar{\alpha}}(H-X) > 0$ and hence

$$\varpi_\alpha(H_2) = \varpi_{\bar{\alpha}}(H) > \varpi_{\bar{\alpha}}(X) \geq c_2 \quad . \qquad \square$$

The modified truncation was already used in Lecture 5. No modification has to be made in Lecture 6, in particular the $\hat{\tau}_P$ in that lecture should not be modified. In Lectures 7 and 8 one has to substitute ${}_{\varepsilon}\sigma_1^2$ for σ_1^2 and the only slight change is in the proof of Lemma 7.3 (essentially the same as Lemma 4.2.2).

9.2. The convex hull of some "orthogonal sets."

The aim of this section is to prove Lemma 5.3.5. Given $s \in \Omega$ we introduced

$$H_s(\mathfrak{a}, T) = s^{-1}(T - H(w_s x)) \quad .$$

LEMMA 9.2.1. *Given* s *and* t *in* Ω *and* T *sufficiently regular*

$$H_s(x, T) - H_t(x, T)$$

is a positive linear combination of the roots γ *such that* $s\gamma > 0$ *and* $t\gamma < 0$.

Let $y = w_s x$ and $\sigma = ts^{-1}$, we have

$$\begin{aligned} H_s(x, T) - H_t(x, T) &= s^{-1}(T - H(y) - H_\sigma(y, T)) \\ &= s^{-1}(T - \sigma^{-1}T + H(w_\sigma n)) \end{aligned}$$

if $y = ank$ with $a \in \mathbf{M}_0$, $n \in \mathbf{N}_0$ and $k \in K$. Using Lemma 6.3 and the comments preceding it we see that this equals

$$s^{-1}\left(\sum_{\substack{\beta > 0 \\ \sigma\beta < 0}} h_\beta \check{\beta} \right) = \sum_{\substack{s\gamma > 0 \\ t\gamma < 0}} h_{s\gamma} \check{\gamma}$$

with $h_\beta = h_{s\gamma} > 0$ if T is sufficiently regular. □

Let P_1 be an ε-invariant parabolic subgroup of G; the real vector space $(\mathfrak{a}_1^G)^\varepsilon$ isomorphic to $\mathfrak{z}^\varepsilon \backslash \mathfrak{a}_1^\varepsilon$ will be denoted by V_1. Any root β of A_1 in N_1 defines a hyperplane $V_{\overline{\beta}}$ which depends only on the orbit $\overline{\beta}$ of β under E. The chambers are the connected components of V_1' the complement in V of the union of the $V_{\overline{\beta}}$. The positive chamber C_1^ε is defined by the following inequalities:

$$\overline{\alpha}(H) > 0 \qquad \forall \alpha \in \Delta_0 - \Delta_0^1 \ .$$

Given $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ there is a standard ε-invariant parabolic subgroup P_2 such that $s(\mathfrak{a}_1^\varepsilon) = \mathfrak{a}_2^\varepsilon$, we define $C_1^\varepsilon(s)$ to be $s^{-1}(C_2^\varepsilon)$ where C_2^ε is the positive chamber of V_2. The chamber $C_1^\varepsilon(s)$ is the set of $H \in V_1$ such that

$$\gamma(H) > 0 \qquad \forall \gamma \in \Delta_1(s, \varepsilon)$$

where $\Delta_1(s, \varepsilon)$ is the set of restrictions to V_1 of the elements in $s^{-1}(\Delta_0 - \Delta_0^2)$.

LEMMA 9.2.2. <u>The map</u> $s \longrightarrow C_1^\varepsilon(s)$ <u>is a bijection between</u> $\Omega(\mathfrak{a}_1^\varepsilon)$ <u>and the set of chambers in</u> V_1.

Given a chamber C^ε in V_1 there is a unique chamber C in $\mathfrak{a}_1^G$ which contains C^ε, and there is at least one $s \in \Omega$ such that $F = sC$ is a "facette" of C_0 the positive Weyl chamber in $\mathfrak{a}_0^G$. Since Ω acts simply transitively on the set of Weyl chambers the "facette" F depends only on C^ε. Since Ω^{M_1} acts trivially on $\mathfrak{a}_1^G$ we may choose

s so that $s\alpha > 0$ for any $\alpha \in \Delta_0^1$. Under those conditions s is uniquely determined by C^ε. Since P_1 is ε-invariant we see that s and $\varepsilon s \varepsilon^{-1}$ have the same properties and hence $s = \varepsilon s \varepsilon^{-1}$. This implies $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ and we conclude that $C^\varepsilon = C_1^\varepsilon(s)$ for a unique $s \in \Omega(\mathfrak{a}_1^\varepsilon)$. □

Two chambers $C_1^\varepsilon(s)$ and $C_1^\varepsilon(t)$ are said to be adjacent if there exist a linear form λ on V_1 unique up to scalars such that λ is positive on $C_1^\varepsilon(s)$ and negative on $C_1^\varepsilon(t)$; in particular the roots γ such that $s\gamma > 0$ and $t\gamma < 0$ have restrictions to V_1 equal to $c_\gamma \lambda$ with $c_\gamma > 0$; the projection $\check{\gamma}_1$ on V_1 of $\check{\gamma}$ equals $c'_\gamma \check{\lambda}$ with $c'_\gamma > 0$.

Given $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ we define H_s^ε to be the projection on V_1 of $H_s(x, T)$. $\sim s^{-1}\Gamma$

LEMMA 9.2.3. _If_ s _and_ t _define adjacent chambers then_

$$H_s^\varepsilon - H_t^\varepsilon = c\check{\lambda}$$

with $c > 0$ (_provided_ T _is sufficiently regular_).

According to Lemma 9.2.1 we have

$$H_s^\varepsilon - H_t^\varepsilon = \sum_{\substack{s\gamma>0 \\ t\gamma<0}} h_{s\gamma} \check{\gamma}_1 = \sum h_{s\gamma} c'_\gamma \check{\lambda}$$

with $h_{s\gamma}$ and c'_γ positive. □

Given $s \in \Omega(\mathfrak{a}_1^{\varepsilon})$ we have introduced $\Delta_1(s, \varepsilon)$; let $\hat{\Delta}_1(s, \varepsilon)$ be the set of ϖ_γ the dual basis of the basis defined by the $\check{\gamma}$ with $\gamma \in \Delta_1(s, \varepsilon)$.

LEMMA 9.2.4. _If_ s _and_ t _define adjacent chambers there is a bijection_ $\theta : \Delta_1(s, \varepsilon) \longrightarrow \Delta_1(t, \varepsilon)$ _such that_ $\theta(\beta) = -\beta$ _if_ β _defines the wall between the two chambers and_ $\varpi_{\theta(\gamma)} = \varpi_\gamma$ _if_ $\gamma \neq \beta$.

Let β be the element in $\Delta_1(s, \varepsilon)$ which defines the wall between $C_1^{\varepsilon}(s)$ and $C_1^{\varepsilon}(t)$, then $-\beta \in \Delta_1(t, \varepsilon)$ and we define θ on β by $\theta(\beta) = -\beta$. Let V_β be the hyperplane defined by β and $\gamma \in \Delta_1(s, \varepsilon)$ with $\gamma \neq \beta$; there exist $\gamma_1 \in \Delta_s(t, \varepsilon)$ such that γ and γ_1 have the same projection on V_β and we define $\theta(\gamma)$ to be γ_1. Then clearly $\varpi_{\theta(\gamma)} = \varpi_\gamma$ if $\gamma \neq \beta$. ☐

Let $\Lambda \in V_1'$, the complement of the walls in V_1, and $s \in \Omega(\mathfrak{a}_1^{\varepsilon})$; we define φ_s^{Λ} to be the characteristic function of the set of $H \in V_1$ such that

$$\left.\begin{aligned} \varpi_\gamma(H) &\leq 0 \quad \text{if} \quad \gamma(\Lambda) > 0 \\ \varpi_\gamma(H) &> 0 \quad \text{if} \quad \gamma(\Lambda) < 0 \end{aligned}\right\} \quad \forall \gamma \in \Delta_1(s, \varepsilon) \quad .$$

Let $a(s, \Lambda)$ be the number of $\gamma \in \Delta_1(s, \varepsilon)$ such that $\gamma(\Lambda) < 0$. In Lecture 5 we have introduced functions ${}_{\varepsilon}B_1^s$ on $\mathfrak{a}_0$, which depend only on the projection on V_1. If $\Lambda \in C_1^{\varepsilon}$ the positive chamber in V_1 we have

$${}_{\varepsilon}B_1^s(H) = (-1)^{a(s,\Lambda)} \varphi_s^{\Lambda}(H) \quad .$$

We now introduce

$$\psi(\Lambda, H) = \sum_{s \in \Omega(\mathfrak{a}_1^{\varepsilon})} (-1)^{a(s,\Lambda)} \varphi_s^{\Lambda}(H-H_s^{\varepsilon}) \quad .$$

Lemma 5.3.5 is an immediate consequence of the

PROPOSITION 9.2.5. The function $H \longrightarrow \psi(\Lambda, H)$ is independent of $\Lambda \in V_1'$ and is the characteristic function of the convex hull of the H_s^{ε} (provided T is sufficiently regular).

We need some more lemmas.

LEMMA 9.2.6. Assume $\varpi_{\gamma}(H-H_s^{\varepsilon}) \leq 0$ for each $\gamma \in \Delta_1(s, \varepsilon)$ and each $s \in \Omega(\mathfrak{a}_1^{\varepsilon})$, then for any $\Lambda \in V_1'$ one has $\psi(\Lambda, H) = 1$.

Given Λ, there is one and only one $s \in \Omega(\mathfrak{a}_1^{\varepsilon})$ such that $\Lambda \in C_1^{\varepsilon}(s)$ and by definition of φ_t^{Λ} we see that $\varphi_t^{\Lambda}(H-H_t^{\varepsilon}) = 0$ unless $t = s$ and hence

$$\psi(\Lambda, H) = \varphi_s^{\Lambda}(H-H_s) = 1 \quad .$$

□

LEMMA 9.2.7. Assume $\psi(\Lambda, H) \neq 0$ and $\Lambda \in C_1^{\varepsilon}(s)$ then provided T is sufficiently regular we have

$$\varpi_{\gamma}(H-H_s^{\varepsilon}) \leq 0 \quad \text{for } \underline{\text{all}} \quad \gamma \in \Delta_1(s, \varepsilon) \quad .$$

Since $\Lambda \longrightarrow \psi(\Lambda, H)$ is constant on $C_1^{\varepsilon}(s)$ it suffices to prove that $\langle \Lambda, H-H_s \rangle \leq 0$.

If $\psi(\Lambda, H) \neq 0$, then for at least one $t \in \Omega(\mathfrak{a}_1^\varepsilon)$ we have ${}^\Lambda_t(H-H_t^\varepsilon) = 1$ and hence

$$\sum \gamma(\Lambda)\varpi_\gamma(H-H_t^\varepsilon) = \left\langle \Lambda, H-H_t^\varepsilon \right\rangle \leq 0 \quad ;$$

but

$$\left\langle \Lambda, H-H_s^\varepsilon \right\rangle = \left\langle \Lambda, H-H_t^\varepsilon \right\rangle + \left\langle \Lambda, H_t^\varepsilon - H_s^\varepsilon \right\rangle$$

and Lemma 9.2.1 implies

$$\left\langle \Lambda, H_t^\varepsilon - H_s^\varepsilon \right\rangle \leq 0 \quad . \qquad \square$$

LEMMA 9.2.8. <u>The set</u> C <u>of</u> $H \in V_1$ <u>such that</u> $\varpi_\gamma(H-H_s^\varepsilon) \leq 0$ <u>for all</u> $\gamma \in \Delta_1(s, \varepsilon)$ <u>and all</u> $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ <u>is the convex hull of the set of</u> H_s^ε <u>with</u> $s \in \Omega(\mathfrak{a}_1^\varepsilon)$, <u>provided</u> T <u>is sufficiently regular</u>.

The set C is an intersection of closed convex sets and hence is a closed convex set. Thanks to Lemma 9.2.1 we see that $H_s^\varepsilon \in C$ if T is sufficiently regular. Now consider $H \in V_1$ outside the convex hull of the H_s^ε, then there exist $\Lambda \in V_1$ such that

$$\left\langle \Lambda, H \right\rangle > \left\langle \Lambda, H_s \right\rangle$$

for all $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ and in particular if s is such that Λ lies in the closure of $C_1^\varepsilon(s)$. This implies $H \notin C$. $\square$

The function $\Lambda \longrightarrow \psi(\Lambda, H)$ is clearly locally constant on V_1'.

To finish the proof of Proposition 9.2.5 we need only to prove that $\psi(\Lambda_\sigma, H) = \psi(\Lambda_\tau, H)$ when $\Lambda_\sigma \in C_1^\varepsilon(\sigma)$ and $\Lambda_\tau \in C_1^\varepsilon(\tau)$ are elements in two adjacent chambers. Let λ be a linear form defining the wall V_λ between the two chambers.

Given $s \in \Omega(\mathfrak{a}_1^\varepsilon)$ then $\varphi_s^{\Lambda_\sigma}(H) = \varphi_s^{\Lambda_\tau}(H)$ if $\gamma(\Lambda_\sigma)\gamma(\Lambda_\tau) > 0$ for all $\gamma \in \Delta_1(s, \varepsilon)$. If it is not so there is one and only one root $\beta \in \Delta_1(s, \varepsilon)$ proportional to λ such that $\beta(\Lambda_\sigma)\beta(\Lambda_\tau) < 0$, and there exist $t \in \Omega(\mathfrak{a}_1^\varepsilon)$ such that $C_1^\varepsilon(s)$ and $C_1^\varepsilon(t)$ are adjacent and separated by V_λ. Since $\gamma(\Lambda_\sigma)\cdot\gamma(\Lambda_\tau) > 0$ if $\gamma \neq \beta$ we see that

$$\xi_s(H) = \varphi_s^{\Lambda_\sigma}(H-H_s^\varepsilon) + \varphi_s^{\Lambda_\tau}(H-H_s^\varepsilon)$$

is the characteristic function of the set of H such that

$$\varpi_\gamma(H-H_s^\varepsilon) > 0 \quad \text{if} \quad \gamma(\Lambda_\sigma) < 0$$

$$\varpi_\gamma(H-H_s^\varepsilon) \leq \quad \text{if} \quad \gamma(\Lambda_\sigma) > 0$$

for all $\gamma \in \Delta_1(s, \varepsilon)$ and $\gamma \neq \beta$. Let θ be the bijection of Lemma 9.2.4; using Lemma 9.2.3 we see that given $\gamma \in \Delta_1(s, \varepsilon)$,

$$\varpi_\gamma(H-H_s) = \varpi_{\theta(\gamma)}(H-H_t)$$

whenever $\gamma \neq \beta$ and hence

$$\xi_s(H) = \xi_t(H)$$

since $\gamma(\Lambda_\sigma)\cdot\theta(\gamma)(\Lambda_\sigma) > 0$ if $\gamma \neq \beta$. This implies that

$$(-1)^{a(s,\Lambda_\sigma)} \varphi_s^{\Lambda_\sigma}(H-H_s) + (-1)^{a(t,\Lambda_\sigma)} \varphi_t^{\Lambda_\sigma}(H-H_t)$$

$$= (-1)^{a(s,\Lambda_\tau)} \varphi_s^{\Lambda_\tau}(H-H_s) + (-1)^{a(t,\Lambda_\tau)} \varphi_t^{\Lambda_\tau}(H-H_t)$$

and hence $\psi(\Lambda_\sigma, H) = \psi(\Lambda_\tau, H)$. □

Let Λ be in any chamber, then

$$v_1^\varepsilon(x, T) = \int_{V_1} \psi(\Lambda, H)\,dH = \lim_{\substack{t\to 0\\ t>0}} \int_{V_1} e^{t\langle\Lambda,H\rangle} \psi(\Lambda, H)\,dH$$

$$= \lim_{\substack{t\to 0\\ t>0}} \sum_{s\in\Omega(\mathfrak{a}_1^\varepsilon)} \int_{V_1} e^{t\langle\Lambda,H\rangle} (-1)^{a(s,\Lambda)} \varphi_s^\Lambda(H-H_s)\,dH \quad .$$

An elementary computation yields

$$v_1^\varepsilon(x, T) = \lim_{t\to 0} \sum_{s\in\Omega(\mathfrak{a}_1^\varepsilon)} c_s \frac{e^{t\langle\Lambda,H_s^\varepsilon\rangle}}{t^{a_1^\varepsilon} \prod_{\gamma\in\Delta_1(s,\varepsilon)} \gamma(\Lambda)}$$

where $c_s = |\det(\gamma_i, \gamma_j)|^{\frac{1}{2}}$, $\gamma_i \in \Delta_1(s, \varepsilon)$. Using Lemma 9.2.4 one shows that $c_s = c_1$ is independent of s and depends only on P_1. Finally we get

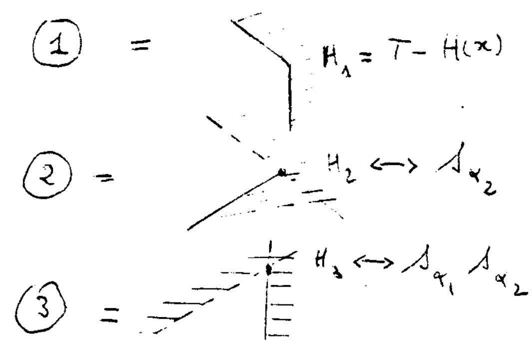
$$v_1^\varepsilon(x, T) = \frac{c_1}{(a_1^\varepsilon)!} \sum_{s\in\Omega(\mathfrak{a}_1^\varepsilon)} \frac{\langle\Lambda,H_s(x,T)\rangle^{a_1^\varepsilon}}{\prod_{\gamma\in\Delta_1(s,\varepsilon)} \gamma(\Lambda)}$$

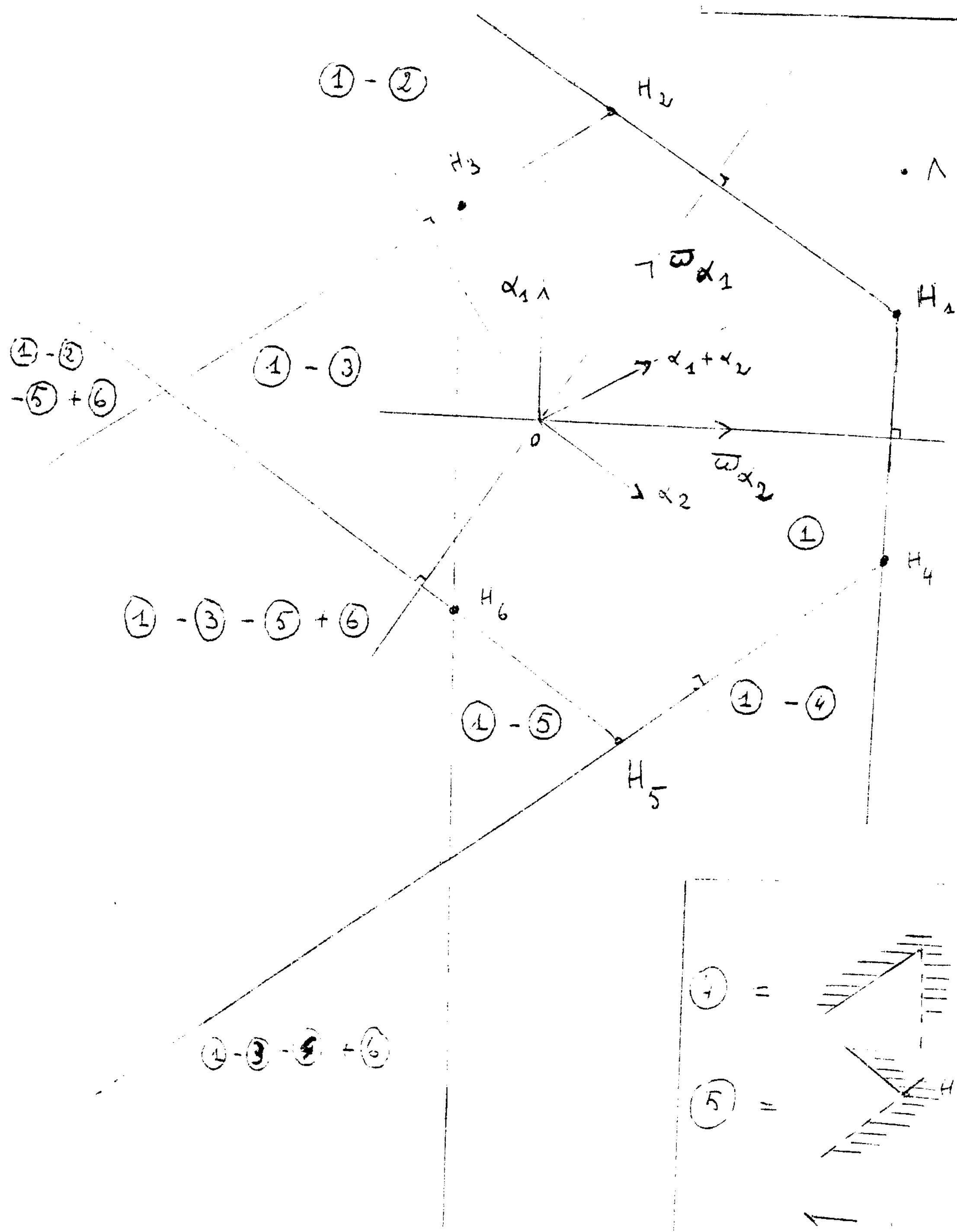
In particular it is a polynomial of T of degree a_1^ε.

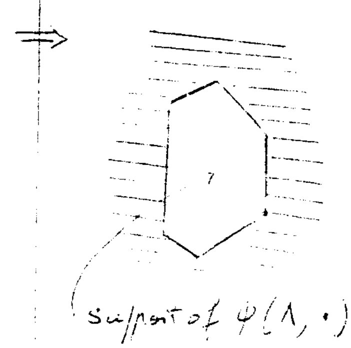
$\Psi(\Lambda,\cdot) = \sum (-1)^{u(s,\Lambda)} \varphi_s^\Lambda(\cdot)$

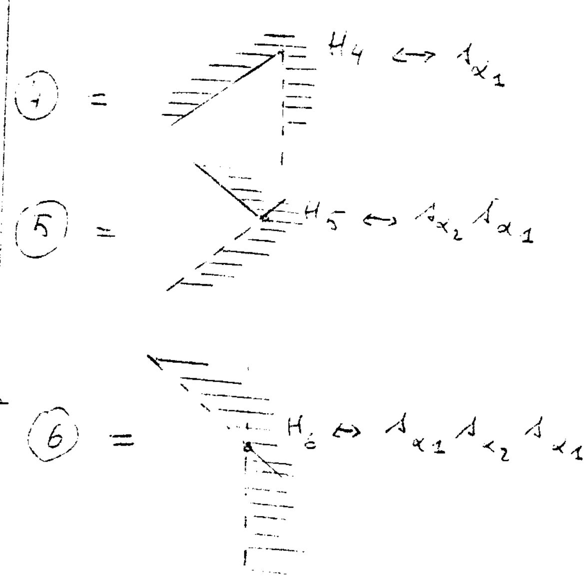
<u>Special case</u> $G = SL_3$ $P_1 = P_0$

$\Lambda \in C_0$, $\varepsilon = 1$









$H_i = s_i^{-1}(T - H(w_{s_i} x))$

$\longleftrightarrow s_i \longleftrightarrow \{ H \to \varphi^\Lambda_{s_i}(H - H_i) \}$