

Undated

Bill,

Since you can answer (one hopes) all questions about the unramified principal series, let me remind you of a question I asked a long time ago.

1.  $G$  - Chevalley group over non-archimedean local field
2.  $G^{\widehat{0}}$  - connected component of associative group
3.  $\widehat{\mathfrak{g}}$  - Lie algebra of  $G^{\widehat{0}}$ .

Consider pairs  $\varphi = \{\widehat{t}, X\}$  up to conjugacy,  $\widehat{t} \in G^{\widehat{0}}$ ,  $\widehat{t}$  semi-simple.  $X \in \widehat{\mathfrak{g}}$ ,  $X$  unipotent.  $\text{Ad } \widehat{t}(X) = qX$ ,  $q$  is the number of elements in the residue field.

Can one associate to each such pair  $\varphi$  a finite non-empty set  $\Pi_{\varphi}$  of irreducible representations of  $G$ , which occur as constituents of the unramified principal series such that

- (i) The sets  $\Pi_{\varphi}$  are disjoint and exhaust the irreducible representations occurring in the principal series
- (ii) The sets  $\Pi_{\varphi}$  are square integrable  $\iff$  there is no proper parabolic subgroup whose Levi factor  $M$  contains  $t$  while its Lie algebra  $\mathcal{M}$  contains  $X$ . (A generalization of indecomposable representation.)
- (iii) If  $\xi$ , taking  $T(F)$ , the split Cartan subgroup, to  $\widehat{L}$  is defined by

$$|\lambda(t)| = g^{(\text{There is perhaps a minus sign here.})\langle \lambda, \widehat{\lambda} \rangle} \quad \widehat{\lambda} = \xi(t)$$

and if  $\chi$  is the character of  $T(F)$  defined by

$$\chi(t) = \widehat{\lambda}(\widehat{t}) \quad \widehat{\lambda} = \xi(t) \quad \varphi = \{\widehat{t}, X\}$$

then each element of  $\Pi_{\varphi}$  is a constituent of  $PS(\chi)$

- (iv) In  $\text{SL}(2)$ , there is such a pair  $\widehat{t} = \begin{pmatrix} q^{1/2} & 0 \\ 0 & q^{-1/2} \end{pmatrix}$ ,  $X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ . If one imbeds  $\text{SL}(2)$  in  $G^{\widehat{0}}$  as a principal three-dimensional subgroup  $\Pi_{\varphi}$  should consist of the special representation alone.

Bob

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