Is a Graviton Detectable?

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1. Introduction

It is generally agreed that a gravitational field exists, satifying Einstein's equations of general relativity, and that gravitational waves traveling at the speed of light also exist. The observed orbital shrinkage of the double pulsar [Weisberg and Taylor, 2005] provides direct evidence that the pulsar is emitting gravitational waves at the rate predicted by the theory. The LIGO experiment now in operation is designed to detect kilohertz gravitational waves from astronomical sources. LIGO has not yet detected a signal, but nobody doubts that gravitational waves are in principle detectable.

This talk is concerned with a different question, whether it is in principle possible to detect individual gravitons, or in other words, whether it is possible to detect the quantization of the gravitational field. The words "in principle" are ambiguous. The meaning of "in principle" depends on the rules of the game that we are playing. If we assert that detection of a graviton is in principle impossible, this may have three meanings. Meaning (a): We can prove a theorem asserting that detection of a graviton would contradict the laws of physics. Meaning (b): We have examined a class of possible graviton detectors and demonstrated that they cannot work. Meaning (c): We have examined a class of graviton detectors and demonstrated that they cannot work in the environment provided by the real universe. We do not claim to have answered the question of "in principle" detectability according to meaning (a). In Section 3 we look at detectors with the LIGO design, detecting gravitational waves by measuring their effects on the geometry of space-time, and conclude that they cannot detect gravitons according to meaning (b). In Sections 4 and 5 we look at a different class of detectors, observing the interactions of gravitons with individual atoms, and conclude that they cannot detect gravitons according to meaning (c). In Sections 6 and 7 we look at a third class of detectors, observing the coherent transitions between graviton and photon states induced by an extended classical magnetic field, and do not reach any definite conclusion.

This paper is a report of work in progress, not a finished product. It raises the question of the observability of gravitons but does not answer it. There is much work still to do.

2. The Bohr-Rosenfeld Argument

Before looking in detail at graviton detectors, I want to discuss a general theoretical question. In 1933 a famous paper by Niels Bohr and Leon Rosenfeld, [Bohr and Rosenfeld, 1933], was published in the proceedings of the Danish Academy of Sciences with the title, "On the Question of the Measurability of the Electromagnetic Field Strengths". An English translation by Bryce de Witt, dated 1960, is in the library at the Institute for Advanced Study in Princeton, bound in an elegant hard cover. This paper was a historic display of Bohr's way of thinking, expounded in long and convoluted German sentences. Rosenfeld was almost driven crazy, writing and rewriting fourteen drafts before Bohr was finally satisfied with it. The paper demonstrates, by a careful and detailed study of imaginary experiments, that the electric and magnetic fields must be quantum fields with the commutation relations dictated by the theory of quantum electrodynamics. The field-strengths are assumed to be measured by observing the motion of massive objects carrying charges and currents with which the fields interact. The massive objects are subject to the rules of ordinary quantum mechanics which set limits to the accuracy of simultaneous measurement of positions and velocities of the objects. Bohr and Rosenfeld show that the quantum-mechanical limitation of measurement of the motion of the masses implies precisely the limitation of measurement of the field-strengths imposed by quantum electrodynamics. In other words, it is mathematically inconsistent to have a classical electromagnetic field interacting with a quantum-mechanical measuring apparatus.

A typical result of the Bohr-Rosenfeld analysis is their equation (58),

$$\Delta E_x(P)\Delta E_x(Q) \sim \hbar |A(P,Q) - A(Q,P)|. \tag{1}$$

Here the left side is the product of the uncertainties of measurement of two averages of the x-component of the electric field, averaged over two space-time regions P and Q. On the right side, $A(P, Q)$ is the double average over regions P and Q of the retarded electric field produced in Q by a unit dipole charge in P. They deduce (1) from the standard Heisenberg uncertainty relation obeyed by the measuring apparatus. The result (1) is precisely the uncertainty relation implied by the commutation rules of quantum electrodynamics. Similar results are found for other components of the electric and magnetic fields.

The question that I am asking is whether the argument of Bohr and Rosenfeld applies also to the gravitational field. If the same argument applies, then the gravitational field must be a quantum field and its quantum nature is in principle observable. However, a close inspection of the Bohr-Rosenfeld argument reveals a crucial feature of their measurement apparatus that makes it inapplicable to gravitational fields. In the last paragraph of Section 3 of the Bohr-Rosenfeld paper, they write: "In order to disturb the electromagnetic field to be measured as little as possible during the presence of the test body system, we shall imagine placed beside each electric or magnetic component particle another exactly oppositely charged neutralizing particle". The neutralizing particles have the following function. Suppose we have a mass carrying a charge or current J whose movement is observed in order to measure the local electric or magnetic field. The movement of the charge or current J produces an additional electromagnetic field that interferes with the field that we are trying to measure. So we must compensate the additional field by adding a second mass, carrying the charge or current $-J$ and occupying the same volume as the first mass. The second mass is constrained by a system of mechanical linkages and springs to follow the movement of the first mass and cancels the fields generated by the first mass. This cancellation is an essential part of the Bohr-Rosenfeld strategy. It is then immediately obvious that the strategy fails for measurement of the gravitational field. The test-objects for measuring the gravitational field are masses rather than charges, and there exist no negative masses that could compensate the fields produced by positive masses.

The conclusion of this argument is that the Bohr-Rosenfeld analysis does not apply to the gravitational field. This does not mean that the gravitational field cannot be quantized.

It means only that the quantization of the gravitational field is not a logical consequence of the quantum behavior of the measuring apparatus. The fact that the electromagnetic field must be quantized does not imply that the gravitational field must be quantized.

3. Can LIGO Detect a Graviton?

In the LIGO experiment, if it is successful, we shall detect a classical gravitational wave, not an individual quantum of gravity. A classical wave may be considered to be a coherent superposition of a large number of gravitons. LIGO is supposed to detect a wave with a strain amplitude f of the order of 10^{-21}. According to [Landau and Lifshitz, 1975], page 370, the energy density of this wave is

$$E = (c^2/32\pi G)\omega^2 f^2, \tag{2}$$

where G is Newton's constant of gravitation and ω is the angular frequency. For a wave with angular frequency 1 Kilohertz and amplitude 10^{-21}, Eq. (2) gives an energy density of roughly 10^{-10} ergs per cubic centimeter. A single graviton of a given angular frequency ω cannot be confined within a region with linear dimension smaller than the reduced wavelength (c/ω). Therefore the energy density of a single graviton of this frequency is at most equal to the energy of the graviton divided by the cube of its reduced wave-length, namely

$$E_s = (\hbar\omega^4/c^3). \tag{3}$$

For an angular frequency of 1 Kilohertz, the single graviton energy density is at most 3.10^{-47} ergs per cubic centimeter. So any gravitational wave detectable by LIGO must contain at least 3.10^{37} gravitons. This wave would be barely detectable by the existing LIGO. For a LIGO apparatus to detect a single graviton, its sensitivity would have to be improved by a factor of the order of 3.10^{37}. Even this vast improvement of sensitivity would probably not be sufficient, because the detection of weak signals is usually limited not only by the sensitivity of the apparatus but also by the presence of background noise. But to see whether detection of single gravitons is possible in principle, we disregard the problem of background radiation and analyze the structure and operation of a super-sensitive LIGO detector.

For a rough estimate of the sensitivity of a LIGO apparatus required to detect a single graviton, we equate (2) with (3). This gives the strain f to be detected by the apparatus,

$$f = (32\pi)^{1/2}(L_p\omega/c), \tag{4}$$

where L_p is the Planck length

$$L_p = (G\hbar/c^3)^{1/2} = 1.4 \times 10^{-33} \text{ cm}. \tag{5}$$

The strain is derived from a measurement of the variation of distance between two masses separated by a distance D. The variation of the measured distance is equal to fD, so long as D does not exceed the reduced wave-length (c/ω) of the graviton. For optimum detectability we take D equal to (c/ω). Then the variation of distance is by (4)

$$\delta = (32\pi)^{1/2}L_p. \tag{6}$$

Up to a factor of order unity, the required precision of measurement of the separation between the two masses is equal to the Planck length, and is independent of the frequency of the graviton.

Is it possible in principle for a LIGO apparatus to measure distances between macroscopic objects to Planck-length accuracy? The following simple arguments give a negative answer to this question. First consider the case in which the objects are floating freely in space. The Heisenberg uncertainty relation between position and momentum of freely floating objects gives the lower bound

$$M\delta^2 \geq \hbar T, \tag{7}$$

for the variation of distance δ, where M is the mass of each object and T is the duration of the measurement. Now T must be greater than the time (D/c) required to communicate between the two masses. If δ is equal to the Planck length, (5) and (7) imply

$$D \leq (GM/c^2). \tag{8}$$

So the separation between the two objects is less than the Schwarzschild radius of each of them, the negative gravitational potential pulling them together is greater than Mc^2, and they are bound to collapse into a black hole before the measurement can be completed.

We next consider the situation that arises when the two masses are clamped in position by a rigid structure. In this case the precision of measurement of the distance between the two objects is limited by quantum fluctuations of the rigid structure. We use a simple dimensional argument to estimate the magnitude of the fluctuations. Let s be the velocity of sound in the structure, let D be the separation between the objects, and let M be the mass of the structure. There will be at least one mode of sound-vibration of the structure which gives a displacement affecting the measurement of D. The mean-square quantum fluctuation amplitude of the displacement in this mode will then be, up to a factor of order unity, at least as large as the zero-point fluctuation,

$$\delta^2 \geq (\hbar D/Ms). \tag{9}$$

The duration of the measurement must be of the order of (D/c), the time it takes the graviton to travel through the apparatus. This duration is shorter than the period (D/s) of the sound-vibration, since s cannot exceed c. Therefore the uncertainty of the measurement is at least equal to the instantaneous vibration-amplitude δ. If the uncertainty is as small as the Planck length (5), then (9) implies

$$(GM/c^2) \geq (c/s)D > D. \tag{10}$$

Again we see that the separation between the two masses is smaller than the Schwarzchild radius of the apparatus, so that the negative gravitational potential of the two masses is greater than Mc^2 and the apparatus will collapse into a black hole. It appears that Nature conspires to forbid any measurement of distance with error smaller than the Planck length. And this prohibition implies that detection of single gravitons with an apparatus resembling LIGO is impossible.

It is clear from Eq. (3) that we have a better chance of detecting a single graviton if we raise the frequency into the optical range and use a different kind of detector. When the frequency is of the order of 10^{15} Hertz or higher, a single graviton can kick an electron out of an atom, and the electron can be detected by standard methods of atomic or particle physics. We are then dealing with the gravito-electric effect, the gravitational analog of the photo-electric effect which Einstein used in 1905, [Einstein, 1905], to infer the existence of quanta of the electromagnetic field, the quanta which were later called photons. The possibility of detecting individual gravitons in this way depends on two quantities, (a) the cross-section for interaction of a graviton with an atom, and (b) the intensity of possible natural or artificial sources of high-frequency gravitons. Most of this talk will be concerned with estimating these two quantities.

4. Graviton Detectors

The simplest kind of graviton detector is an electron in an atom, which we may approximate by considering the electron to be bound in a fixed non-relativistic potential $V(r)$. We choose coordinate axes so that the z-axis is the direction of motion of a graviton. There are then two orthogonal modes of linear polarization for the graviton, one with the wave-amplitude proportional to xy, and the other with the amplitude proportional to $(x^2 - y^2)$. We choose the x and y-axes so that they make angles of 45 degrees to the plane of polarization of the graviton. Then the matrix element for the electron to absorb the graviton and move from its ground state a to another state b is proportional to the mass-quadrupole component,

$$D_{ab} = m \int \psi_b^* xy \psi_a d\tau, \tag{11}$$

where m is the electron mass. Eq. (11) is the quadrupole approximation, which is valid so long as the wave-length of the graviton is large compared with the size of the atom. The total cross-section for absorption of the graviton by the electron is

$$\sigma(\omega) = (4\pi^2 G\omega^3/c^3) \sum_b |D_{ab}|^2 \delta(E_b - E_a - h\omega), \tag{12}$$

where E_a and E_b are the energies of the initial and final states. It is convenient to consider a logarithmic average of the cross-section over all frequencies ω,

$$S_a = \int \sigma(\omega) d\omega/\omega. \tag{13}$$

Integration of (12) gives the sum-rule

$$S_a = 4\pi^2 L_p^2 Q, \tag{14}$$

where the Planck length L_p is given by (4), and

$$Q = \int |(x\partial/\partial y + y\partial/\partial x)\psi_a|^2 d\tau \tag{15}$$

is a numerical factor of order unity. It is remarkable that the average cross-section (14) is independent of the electron mass and of the nuclear charge. The same formula (14) holds for the absorption of a graviton by a neutron or proton bound in a nuclear potential.

For simplicity we assume that the electron is in an s-state with a wave-function $f(r)$ which is a function of distance r from the nucleus. Then (15) becomes

$$Q = (\int r^4 [f'(r)]^2 dr)/(3 \int r^2 [f(r)]^2 dr). \tag{15}$$

The inequality

$$\int r^4 [f' + (3/2r) f]^2 dr > 0 \tag{16}$$

implies that for any $f(r)$

$$Q > 3/4. \tag{17}$$

On the other hand, if

$$f(r) = r^{-n} \exp(-r/R), \tag{18}$$

then

$$Q = 1 - (n/6). \tag{19}$$

From (17) and (19) it appears that for any tightly-bound s-state Q will be close to unity. The cross-section for absorption of a graviton by any kind of particle will be of the same magnitude

$$4\pi^2 L_p^2 = 4\pi^2 G\hbar/c^3 = 8 \times 10^{-65}\ \mathrm{cm}^2, \tag{20}$$

spread over a range of graviton energies extending from the binding-energy of the particle to a few times the binding-energy. For any macroscopic detector composed of ordinary matter, the absorption cross-section will be of the order of 10^{-41} square centimeters per gram.

5. Thermal Graviton Generators

We have a splendid natural generator of thermal gravitons with energies in the kilovolt range, producing far more gravitons than any artificial source is likely to generate. It is called the sun. Stephen Weinberg long ago calculated [Weinberg, 1965] the graviton luminosity of the sun, caused by gravitational bremsstrahlung in collisions of electrons and ions in the sun's core. A later calculation [Gould, 1985] corrected a mistake in Weinberg's paper but does not substantially change the result. For an electron-ion collision with energy E, the differential cross-section $p(\omega)$ for producing a graviton of energy $\hbar\omega$ is divergent at low energies, so that the total cross-section has no meaning. The physically meaningful quantity is the integral of the differential cross-section multiplied by the energy of the graviton,

$$\int p(\omega)\hbar\omega d\omega = (320/9) Z^2 \alpha^2 L_p^2 E, \tag{21}$$

where α is the electromagnetic fine-structure constant and Z is the charge of the ion. Including a similar contribution from electron-electron collisions, (21) gives a total graviton luminosity of the sun

$$L_g = 79\ \text{Megawatts}, \tag{22}$$

or about 10^{24} gravitons per second with energy in the kilovolt range. This gives a flux at the earth of

$$F_g = 4 \times 10^{-4} \text{ gravitons per cm}^2 \text{ per second.} \tag{23}$$

If we imagine the whole mass of the earth to be used as a graviton detector, with the cross-section (20) per electron and the flux (23), the counting-rate is 2.4×10^{-17} per second. If the experiment continues for the life-time of the sun, which is 5 billion years, the expected total number of gravitons detected will be 4. The experiment barely succeeds, but in principle it can detect gravitons.

According to [Gould, 1985], there exist in the universe sources of thermal gravitons which are stronger than the sun, namely hot white dwarfs at the beginning of their lives, and hot neutron stars. Gould estimates the graviton luminosities of a typical white dwarf and a typical neutron star to be respectively 10^4 and 10^{10} times solar. Their luminosities are roughly proportional to their central densities. But the life-times during which the stars remain hot are shorter than the life-time of the sun, being of the order of tens of millions of years for the white dwarf and tens of thousands of years for the neutron star. The life-time output of gravitons will therefore be respectively 100 and 10^5 times solar. To stretch the theoretical possibilities of detection to the limit, we may suppose the detector to have mass equal to the sun and to be orbiting around the source of gravitons at a distance of 0.01 astronomical unit with an orbital period of 8 hours. Then the expected number of gravitons detected will be of the order of 10^{13} for the white dwarf and 10^{16} for the neutron star. The detection rate is roughly one per minute for the white dwarf and 3×10^4 per second for the neutron star. The conclusion of this calculation is that graviton detection is in principle possible, if we disregard the problem of discriminating the graviton signal from background noise.

The most important source of background noise is probably the neutrinos emitted by the sun or the white dwarf or the neutron star as the case may be. These neutrinos can mimic graviton absorption events by ejecting electrons from atoms as a result of neutrino-electron scattering. The neutrinos have higher energy than the gravitons, but only a small fraction of the neutrino energy may be transferred to the electron. From the sun, about 10^{14} neutrinos are emitted for each graviton, and the cross-section for neutrino-electron scattering is about 10^{20} times the cross-section for graviton absorption, [see Fukugita and Yanagida, 2003]. Therefore there will be about 10^{34} neutrino background events for each graviton absorption event.

For white-dwarfs and neutron-stars the ratio of background to signal is even larger, since neutrino production and scattering cross-sections increase with temperature more rapidly than graviton production and absorption cross-sections. Without performing detailed calculations, we can assert that for all thermal sources of gravitons the ratio of neutrino background to graviton signal will be of the order of 10^{34} or greater. In all cases, the total number of detected graviton events is vastly smaller than the square-root of the number of background events. The graviton signal will be swamped by the statistical scatter of the background noise.

Before jumping to conclusions about the detectability of gravitons, we must explore possible ways in which the neutrino background events might be excluded. The first possible way is to surround the detector with a shield thick enough to stop neutrinos but let gravitons pass. If the shield is made of matter of ordinary density, its thickness must be of the order 10^{10} kilometers, and its mass is so large that it will collapse into a black hole. The second possible way is to surround the graviton detector with neutrino detectors in anti-coincidence, to catch the outgoing neutrino after each scattering event. This way fails for the same reason as the shield. The neutrino detectors would need to be at least as massive as the shield. The third possible way is to build a shield or a set of anti-coincidence detectors out of some mythical material with super-high density. The known laws of physics give us no clue as to how this might be done. We conclude that, if we are using known materials and known physical processes, detection of thermal gravitons appears to be impossible.

6. Non-thermal Gravitons

It is possible to imagine various ways in which energetic objects such as pulsars may emit non-thermal gravitons of high energy. One such way is a process first identified by

[Gertsenshtein, 1961], the coherent mixing of photon and graviton states in the presence of an extended classical magnetic field. The graviton emission from various celestial objects resulting from the Gertsenshtein process was calculated by [Papini and Valluri, 1989]. Some interestingly high graviton luminosities were predicted.

The Gertsenshtein process results from the interaction energy

$$(8\pi G/c^4)h_{ij}T_{ij}, \tag{24}$$

between the gravitational field h_{ij} and the energy-momentum tensor T_{ij} of the electromagnetic field. This interaction expresses the fact that electromagnetic fields have weight, just like other forms of energy. Now suppose that h_{ij} is the field of a graviton traveling in the z direction and

$$T_{ij} = (1/4\pi)(B_i + b_i)(B_j + b_j), \tag{25}$$

is the energy-momentum of the photon magnetic field b_i superimposed on a fixed classical magnetic field B_i. Then the interaction (24) contains the term

$$I = (4G/c^4)h_{xy}B_x b_y, \tag{26}$$

bilinear in the graviton and photon fields. The effect of this bilinear term is to mix the photon and graviton fields, so that a particle that is created as a photon may be transformed into a graviton and vice versa. There is an oscillation between graviton and photon states, just like the oscillation between neutrino states that causes neutrinos to change their flavors while traveling between the sun and the earth. If a photon travels a distance D though a uniform transverse magnetic field B, it will emerge as a graviton with probability

$$P = \sin^2(G^{1/2}BD/2c^2) = \sin^2(B/L), \tag{27}$$

with the mixing-length

$$L = (2c^2/G^{1/2}B) \tag{28}$$

independent of wave-length. In all practical situations, D will be small compared with L, so that

$$P = (GB^2 D^2 / 4c^4). \tag{29}$$

The quadratic dependence of P on D makes this process interesting as a possible astrophysical source of gravitons. The numerical value of L according to (28) is roughly

$$L = (10^{25}/B), \tag{30}$$

when L is measured in centimeters and B in Gauss.

We may also consider the Gertsenshtein process as the basis of a graviton detector consisting of a hollow pipe of length D filled with a transverse magnetic field B. The tube must be accurately pointed at a putative source of gravitons in the sky. At the far end of the tube is a shield to block incident photons, and at the near end is a detector of photons resulting from the conversion of gravitons on their way through the tube. If D is one astronomical unit (10^{13} cm), then (29) gives

$$P = 10^{-24} B^2. \tag{31}$$

The field B must be very strong to obtain a reasonable rate of conversion of gravitons to photons. A detector with the same design has been used in a real experiment to detect axions that might be created by thermal processes in the core of the sun [Zioutas et al., 2005]. The axion field is supposed to interact with the electromagnetic field with an interaction energy similar to (26), but with a much larger coupling constant. The experimenters at CERN in Switzerland are using a surplus magnet from the Large Hadron Collider project as an axion-detector, pointing it at the sun and looking for kilovolt photons resulting from conversion of axions into photons. The length of the magnet is 9 meters and the magnetic field is 9×10^4 Gauss. They have not yet detected any axions.

The Gertsenshtein process does not require the classical magnetic field to be uniform. For a non-uniform field, the conversion of photons to gravitons still occurs with probability given by (27), if we replace the product BD by the integral of the transverse component of B along the trajectory of the photons. Likewise, the conversion will not be disturbed by a background gravitational field, even when the field is strong enough to curve the photon trajectory, because the gravitational field acts in the same way on photons and gravitons. In a curved space-time, the photons and the gravitons follow the same geodesic paths, and the photon and graviton waves remain coherent.

7. Non-linear Electrodynamics

However, there is an important disturbing factor which was neglected in previous discussions of the Gertsenshtein process. The disturbing factor is the non-linearity of the electromagnetic field caused by quantum fluctuations of electron-positron pairs in the vacuum, [Euler and Heisenberg, 1936; Wentzel, 1943]. The fourth-order term in the electromagnetic field energy density is [Wentzel, 1943, page 190],

$$(\alpha/360\pi^2 H_c^2)[(E^2 - H^2)^2 + 7(E.H)^2], \tag{32}$$

where α is the fine-structure constant and

$$H_c = (m^2c^3/e\hbar) = 5.10^{13}\text{Gauss} \tag{33}$$

is the critical magnetic field at which electron-positron pair fluctuations become noticeable.

When the field in (32) is divided into classical and photon components as in (25), there is a term quadratic in both the classical and photon fields,

$$(\alpha/360\pi^2 H_c^2)(4(B.b)^2 + 7(B.e)^2), \tag{34}$$

where b and e are the magnetic and electric fields of the photon. From (34) it follows that the photon velocity v is not equal to c but is reduced by a fraction

$$g = 1 - (v/c) = (k\alpha B^2/360\pi^2 H_c^2). \tag{35}$$

The coefficient k is equal to 4 or 7 for a photon polarized with its magnetic field or its electric field parallel to B. We consider the case $k = 4$, since that case is more favorable to the Gertsenshtein process. Since the graviton field is not affected by the non-linear electromagnetic interaction (32), the graviton velocity is precisely c, and the photon and graviton waves will lose coherence after traveling for a distance

$$L_c = (c/g\omega) = (90\pi^2 c H_c^2/\alpha B^2\omega) = (10^{43}/B^2\omega). \tag{36}$$

If the propagation distance D is larger than L_c, the Gertsenshtein process fails and the formula (29) for the photon-graviton conversion probability is incorrect. A necessary condition for the Gertsenshtein process to operate is

$$DB^2\omega \leq 10^{43}. \tag{37}$$

Furthermore, even when the Gertsenshtein process is operating, the probability of photon-graviton conversion according to (29) and (37) is

$$P \leq (10^{36}/B^2\omega^2). \tag{38}$$

We are interested in detecting astrophysical sources of gravitons with energies up to 100 kilovolts, which means frequencies up to 10^{20}. With $\omega = 10^{20}$, (37) and (38) become

$$D \leq (10^{23}/B^2), \quad P \leq (10^{-4}/B^2). \tag{39}$$

We consider two situations in which (39) has important consequences. First, with typical values for the magnetic field and linear dimension of a pulsar, $B = 10^{12}$ and $D = 10^6$, (39) shows that the Gertsenshtein process fails by a wide margin. The calculations of the graviton luminosity of pulsars in [Papini and Valluri, 1989] assume that the Gertsenshtein process is producing high-energy gravitons. These calculations, and the high luminosities that they predict, are therefore incorrect. Second, in the hollow pipe graviton detector which we considered earlier, (39) shows that the Gertsenshtein process can operate with a modest field, $B = 10^5$ Gauss, and a pipe length $D = 10^{13}$ cm, but the probability of detection of each graviton traveling through the pipe is only 10^{-14}. If the field is made stronger, the length of the pipe must be shorter according to (39), and the probability of detecting a graviton becomes even smaller.

8. Conclusions

Many papers have been published, for example [Eppley and Hannah, 1977; Page and Geilker, 1981], claiming to demonstrate that the gravitational field must be quantized. What these papers demonstrate is that a particular theory with a classical gravitational field interacting with quantum-mechanical matter is inconsistent. Page and Geilker assume that the classical gravitational field is generated by the expectation value of the energy-momentum tensor of the matter in whichever quantum state the matter happens to be. They performed an ingenious experiment to verify that this assumption in fact gives the wrong answer for a measurement of the gravitational field in a real situation.

In this paper I am not advocating any particular theory of a classical gravitational field existing in an otherwise quantum-mechanical world. I am raising two separate questions. I am asking whether either one of two theoretical hypotheses may be experimentally testable. One hypothesis is that gravity is a quantum field and gravitons exist. A second hypothesis is that the gravitational field is a statistical concept like entropy or temperature, only defined for gravitational effects of matter in bulk and not for effects of individual elementary particles. If the second hypothesis is true, then the gravitational field is not a local field like the electromagnetic field. The second hypothesis implies that the gravitational field at a point in space-time does not exist, either as a classical or as a quantum field.

Now I assert that both of the two hypotheses may or may not be experimentally testable. Analysis of the properties of graviton-detectors, following the methods of this paper, might be able to throw light on both hypotheses. Three outcomes are logically possible. If a graviton detector is possible and succeeds in detecting gravitons, then the first hypothesis is true and the second is false. If a graviton detector is possible and fails to detect gravitons, then the first hypothesis is false and the second is open. If a graviton detector is in principle impossible, then both hypotheses remain open. Even if their existence is not experimentally testable, gravitons may still exist.

The conclusion of our analysis is that we are still a long way from settling the question whether gravitons exist. But the question whether gravitons are in principle detectable is also interesting and may be easier to decide.

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